# Equilibrium Analysis of Simultaneous Demand and Supply

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# Outline

- Main objective
- Brief introduction of an economic demand and supply system
- Full information analysis and limited information analysis
- Example: linear simultaneous demand and supply
- Discrete choice and supply
- Conclusion

# Main Objective

- A comprehensive, yet understandable overview of the classical topic in econometrics
  - Simultaneous analysis of demand and supply is a classical, important and still alive topic in econometrics
  - It covers a wide range of economic theories and statistical techniques
- Illustrate how statistics is used in conjunction with economic theories in econometrics
  - economic concepts are the basis
  - statistics is a tool

# Demand and Supply System

- Demand side: given price, consumers maximize their utilities by choosing consumption subject to budget constraint.

$$max_q u(q, p, x), \ s.t. \ qp \leq I.$$

So the optimal demand $q^*$ is a function of price $p$ and some "exogeneous" variable $x$, $q^* = f_q(p, x)$.

- Supply side: firms maximizes their profit by choosing prices, given demand

$$max_p \pi(q, c, y).$$

So the optimal price $p^*$ is a function of demand $q$ and some "exogeneous" variable $y$, $p^* = f_p(q, y)$.

- In "equilibrium", demand and price must satisfy

$$q^* = f_q(p^*, x), \ p^* = f_p(q^*, y).$$

# Full/Limited Information Analysis

- Economists usually *assume* the observed consumption (demand) and price (supply) are equilibrium quantities.
- Suppose now we observe the consumption and price. Goal: estimate "demand/supply parameters".
  - Full information analysis: use both equations
  - Limited information analysis: use only one equation

# Example: Linear Simultaneous Demand and Supply

- Suppose

$$\begin{aligned} q^* &= \alpha_q + \beta_q \cdot p^* + \epsilon_q \\ p^* &= \alpha_p + \beta_p \cdot q^* + \epsilon_p. \end{aligned}$$

- $\epsilon_q$ and $\epsilon_p$ are usually called demand shock and supply shock, respectively.
- How does the randomness comes in?
- Goal: estiamte demand and supply parameters based on $\{q_t,\ p_t,\ t = 1, 2, ..., T\}$.

# Limited Information Analysis

- Suppose (for some reasons), we only want to investigate the demand side.
- Would running an OLS regression of $q^*$ on $p^*$ yield consistent estimates of $\alpha_q$ and $\beta_q$?
- Endogeneity problem. $p^*$ is also a function of $\epsilon_q$, hence necessarily correlated with $\epsilon_q$.
- This problem is usually called simultaneity problem.

# Limited Information Analysis (Cont.)

- Formally, the statistical model of LIA is

$$q = \alpha_q + \beta_q \cdot p + \epsilon_q, \; E(\epsilon_q) = 0, \; E(p\epsilon_q) \neq 0.$$

- Note: $p$ is also random, different from fixed regressors in design courses.
- GMM (Instrumental variable regression)
  - Find "instruments" that are correlated with $p$ but uncorrelated with $\epsilon_q$.
  - Estimation is based on moment conditions.

# Full Information Analysis

- $p^*$ and $q^*$ could be solved out as functions of $\epsilon_q$ and $\epsilon_p$.
- Likelihood approach. Distributional assumptions are assumed on demand and supply shocks. The joint distribution of $p^*$ and $q^*$ is then induced by the transformation.
- GMM approach.

# Interpretation

- Statistical interpretation of $\beta_q$ and $\beta_p$. What are they?
- Economic interpretation. Remember: they are decuded from some underlying economic model. The parameters have specific meanings in that model.

# Discrete Choice Modeling for Demand

- A popular way to model demand.
- A consumer is assumed to face a choice set consisting $J$ products. The consumer is assumed to consume only one product that gives him/her the highest utility.
- The utility for consumer $i$ buying product $j$ is

$$u_{ij} = x_{ij}'\beta + \alpha \cdot p_j + \xi_j + \epsilon_{ij},$$

  where $x_{ij}$ is observables, $\xi_j$ is the product specific characteristics, $\epsilon_{ij}$ is the "iidiosyncratic" heterogeneity.
- $\xi_j$ and $\epsilon_{ij}$ are unobserved by the researchers.

# Discrete Choice Modeling for Demand (Cont.)

- The probability of choosing product $j$ is

$$Pr\left(u_{ij} \geq u_{i1},\ u_{ij} \geq u_{i2}, ..., u_{ij} \geq u_{iJ}\right).$$

- Conditional on $x_{ij}$, $p_j$ and $\xi_j$, if $\epsilon_{ij}$ is assumed to follow type I extreme value distribution, the above quantity becomes our familiar logistic regression formula:

$$\frac{e^{x'_{ij}\beta+\alpha p_j+\xi_j}}{\sum_k e^{x'_{ik}\beta+\alpha p_k+\xi_k}}.$$

# Discrete Choice Modeling for Demand (Cont.)

- Remember we don't observe $\xi_j$.
- In this type of model, researchers often "belief/assume/argue" price is endogenous, meaning price is correlated with $\xi_j$.
- Why? Intuition is: the optimal price is set according to demand, which means the optimal price is a function of $\xi_j$.
- Using game theory or other theoretical tools, the optimal price is a highly non-linear function of $\xi_j$.

# Full Information Analysis Under Discrete Choice Model

- Want to maximize the joint likelihood of price and demand.

$$
\begin{aligned}
L\left(price,\, demand\right) &= L\left(demand \mid price\right) L(price) \\
&= \left( \int \frac{e^{x_{ij}'\beta+\alpha p_j+\xi_j}}{\sum_k e^{x_{ik}'\beta+\alpha p_k+\xi_k}} f\left(\xi_j|p_j\right) d\xi_j \right) f(price)
\end{aligned}
$$

- Not easy to obtain $f\left(\xi_j|p_j\right)$ !!!

# Full Information Analysis Under Discrete Choice (Cont.)

Even more ugly if $\beta$ is assumed to be random. (random effect)

$$\int \left\{ \left( \int \frac{e^{x_{ij}'\beta+\alpha p_j+\xi_j}}{\sum_k e^{x_{ik}'\beta+\alpha p_k+\xi_k}} f\left(\xi_j|p_j\right) d\xi_j \right) f(price) \right\} f(\beta) d\beta$$

Potential research topic on this. I guess, maybe.....copula???

# Conclusion

- Economic theory underlies all of them.
- This is just one face of econometrics.
- Statistics plays a role there!

References???

Too many.....

email me if you are really interested.

Happy Chinese New Year !!!!

Thank you.