Exercise 1

An experimenter wishes to investigate the effects of three variables - pressure, temperature and type of catalyst - on yield in a refining process. If the experimenter intends to use three settings for temperature, three settings for pressure, and two types of catalysts, how many experimental runs must be conducted to run all possible combinations of pressure, temperature and type of catalyst?

Temperature (experiment 1) – 3 outcomes

Pressure (experiment 2) – 3 outcomes

Catalyst (experiment 3) – 2 outcomes

$\therefore$ Total # possible outcomes $= 3 \times 3 \times 2 = 18$

Exercise 2

How many different telephone numbers can be formed from a seven-digit number if the first digit cannot be zero?

Digit	1	2	3	. . . .	7
# outcomes	$\boxed{9}$	10	10	. . .	10

Cannot be 0, so there are only 9 outcomes

∴ Total # possible numbers $= 9 \times 10^6$

Exercise 3

How many 6-digit serial numbers can be formed if no digit is to be repeated?

Total # possible numbers $= 10 \times 9 \times 8 \times 7 \times 6 \times 5$

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Exercise 4

In how many ways can three executive members of a committee be assigned to three different positions: chair, treasurer, secretary?

Total # possible arrangements = $\underset{\text{Chair}}{3} \times \underset{\text{treasurer}}{2} \times \underset{\text{secretary}}{1} = 6 = 3!$

any 1 of the 3 members can be the chair

any 1 of the 2 members (who are not chair) can be the treasurer

the last person will be the secretary

Exercise 5

In how many ways can a committee of three be selected from among ten people if each committee member takes on a different position (chair, treasurer, secretary)?

Total # possible arrangements $= 10 \times 9 \times 8 = {}_{10}P_3$

$= 720$

Note:

$$\overbrace{\binom{n}{r}}^{\#\text{ subsets of size } r \text{ (unordered)}} \times \overbrace{r!}^{\#\text{ ways of ordering } r \text{ objects}} = \overbrace{{}_nP_r}^{\#\text{ ordered arrangements of } r \text{ objects from } n \text{ objects}}$$

Hence, we have

$$\binom{n}{r} = \frac{{}_nP_r}{r!} = \frac{n!}{r!(n-r)!}$$

Gift card example revisited:

Total number of distinct subsets of two cards $= {}_5C_2 = \frac{5!}{2!\,3!} = 10$

Exercise 6

In how many ways can a committee of three be selected from among ten people? The order in which 3 people are selected from the 10 people does NOT matter, so the total # committees $= {}_{10}C_3 = \frac{10!}{3!\,7!} = 120$

Exercise 7

An assembly operation for a computer circuit board consists of four distinct operations, which can be performed in any order.

(a) In how many ways can the assembly operation be performed?

(b) One of the operations involves soldering wire to a microchip. In how many ways can this particular operation come first or second?

(a) The order is important here as we want to know how many ways the assembly operation can be performed.

$\therefore$ # ways = # permutations of 4 from 4 operations = $4! = 24$

(b) The remaining 3 operations

_ 1 _ _ 2 _ _ 3 _

soldering comes first or second

- For soldering to take place first or second, there are 2 ways
- For the remaining 3 operations, the # of arrangements = $3! = 6$

$\therefore$ Total # ways = $2 \times 6 = 12$

Exercise 8

A fleet of eight taxis is to be divded among three airports, A, B and C, with two going to A, five to B, and one to C.

(a) In how many ways can this be done?

(b) Jones drives one of the taxis. In how many ways will Jones end up at airport C?

(a) The 3 airports are distinct

8 taxis
- airport A $n_1 = 2$
- airport B $n_2 = 5$
- airport C $n_3 = 1$

Total # ways

$$= \frac{8!}{2!\,5!\,1!} = 168$$

(b) To have Jones going to airport C, this would imply the remaining 7 taxis going to the other 2 airports (2 to A and 5 to B)

$$\therefore \text{ # ways } = \frac{7!}{2!\,5!} = 21$$

Exercise 9

In how many ways can 8 people be seated in a row if

Suppose the 8 people are A, B, C, ... H

(a) persons A and B must sit next to each other.

\# ways = # permutations of 7 (treat A, B together as a group) from 7

× # permutations of 2 from 2 (arranging A, B)

$= 7!\,2!$

(b) there are 4 men and 4 women and no 2 men or 2 women can sit next to each other.

W = woman   M = man

one possible arrangement: W M W M W M W M
another possible arrangement: M W M W M W M W

\# ways = (# permutations of 4 from 4 men)

× (# permutations of 4 from 4 women)

× 2 ← W or M comes first

$$= (4!)(4!)(2)$$

(c) there are 4 married couples and each couple must sit together.

$\boxed{W_1 M_1}$ $\boxed{W_2 M_2}$ $\boxed{W_3 M_3}$ $\boxed{W_4 M_4}$

\# ways = (# permutations of 4 (a couple as a group) selected from 4 couples)

× (# permutations of 2 from 2 within each couple)^4

$= (4!)(2^4)$

