Motivation

Ruben Zamar
Department of Statistics
UBC

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LEARNING OUTCOMES:

- Gain an understanding of why to study Probability.
- Look forward to the upcoming lectures.

THE METHODOLOGY OF SCIENCE

There are two methods for deriving new knowledge in science

Deductive resoning
(Aristotle, 384–322 BC)

and

Inductive reasoning
(Hasan Ibn Alhazen, 965-1040)

Examples of deductive reasoning

Example 1: Let A be an angle.

First premise: If $A \in [95^o, 110^o] \Rightarrow A$ is *obtuse*.

Second Premise: $A = 105^o$

Conclusion: A is obtuse (**correct**)

Example 2: Let A be an angle.

First premise: If $A \in [95^o, 110^o] \Rightarrow A$ is *obtuse*.

Second Premise: A is obtuse

Conclusion: $A \in [95^o, 110^o]$ (**wrong.** Why?)

Example 3: Let A be an angle.

First premise: If $A \in [95^o, 110^o] \Rightarrow A$ is *obtuse*.

Second Premise: A is not obtuse

Conclusion: $A \notin [95^o, 110^o]$ (**correct. Why**?)

DEDUCTIVE REASONING

Correct premises	+	Correct logic	=	Correct conclusion

Mathematical Induction

Example: Prove that, for all $n = 1, 2, \ldots$

$$1 + 2 + \cdots + n \quad = \quad \frac{n(n+1)}{2}.$$

First prove the base case: $n = 2$

$$1 + 2 = 3 \quad , \quad 2 \times 3/2 = 3.$$

Induction Step: Assume the formula holds for $n = m$:

$$1 + 2 + \cdots + m = \frac{m(m+1)}{2}$$

Show now that the formula holds for $n = m + 1$

$$
\begin{aligned}
1 + 2 + \cdots + (m + 1) &= (1 + 2 + \cdots + m) + (m + 1) \\
&= m(m + 1)/2 + (m + 1) \\
&= (m + 1)\left[\frac{m}{2} + 1\right] \\
&= (m + 1)\frac{m + 2}{2}.
\end{aligned}
$$

Conclusion. The premise holds for all integer $n \geq 1$.

BASIC STEPS OF THE INDUCTIVE METHOD

1. Make an observation

   Ex: my toaster won't toast the bread

2. Ask a question

   Ex: Why doesn't my toaster toast the bread?

3. Propose a hypothesis or potential answer

   Ex: maybe the outlet is broken.

4. Make a prediction based on the hypothesis

   Ex: If I plug the toaster into a different outlet, then it will toast the bread.

5. Test the prediction

   Ex: plug the toaster into a different outlet and try again.

6. Iterate: use the results to make new hypotheses or predictions

   Ex: My bread toasts! But what is wrong with the outlet?

   My bread still won't toast. Maybe there is a broaken wire in the toaster?

BOILING POINT OF WATER

- **Hypothesis:** Water boils at 212^o F regardless of the altitude.

To test this hypothesis we measure the boiling point of water at different altitudes.

EXPERIMENTAL DATA

ALTITUDE (feet)	TEMP. (F)	ALTITUDE (feet)	TEMP. (F)
0	212	11000	192
1000	210	18000	174
3000	208	26247	165
10000	193	27400	163
10300	195	29035	162

EXPERIMENTAL DATA

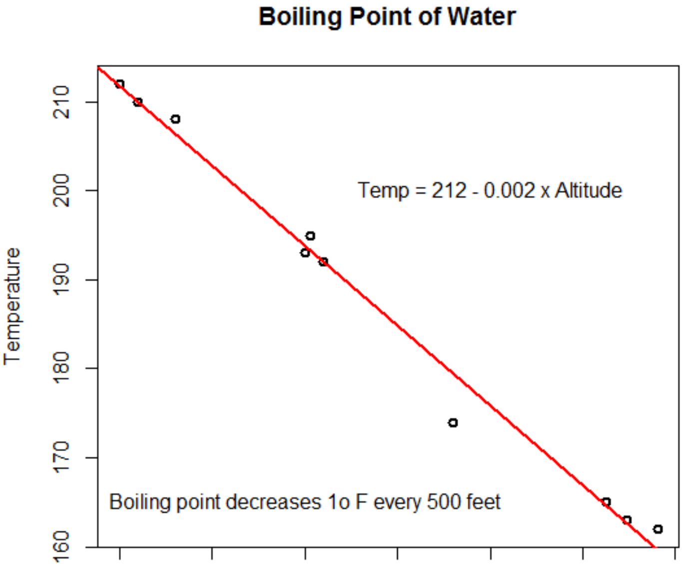
- Measurements are subject to error.

$$\begin{aligned} \text{Measured Altitude} &= \text{True Altitude} + \text{Measurement Error} \\ \text{Measured Temp} &= \text{True Temp} + \text{Measurement Error} \end{aligned}$$

CONCLUSION

- In 8 out of 9 comparisons the measured boiling point is smaller at higher elevation.
- How likely is to observe something like that if the hypothesis of constant boiling point is true?
- Data seems to strongly contradict the hypothesis.
- **Conclusion:** reject the hypothesis of constant boiling point and search for some pattern.

PATTERN


Boiling Point of Water
Temperature
210
200
190
180
170
160
Temp = 212 - 0.002 x Altitude
Boiling point decreases 1o F every 500 feet