STAT 547C

MIDTERM PRACTICE PROBLEMS

Problem 1: Give a concise definition of

- sigma field
- probability function
- probability space
- limsup and liminf of a sequence of events
- random variable
- convergence in probability for a sequence X_n of random variables
- convergence in distribution for a sequence X_n of random variables
- convergence with probability one
- limit of a sequence of events
- State the following results
  - Borel-Cantelli Lemmas
  - Dominated Convergence Theorem

Problem 2: Suppose that A_n is an increasing sequence of nested events (that is, $A_n \uparrow A$). Show that $\lim_{n\to\infty} P(A_n) = P(A)$.

Problem 3: Let

$$g(t) = E\{g(X,t)\}$$

Give sufficient conditions and a proof for the following result

$$\frac{dg(t)}{dt} g(t) \quad = \quad E\left\{\frac{\partial}{\partial t} g(X,t)\right\}$$

Problem 4: Suppose that given (Y,Q), $X \sim N\left(YQ, YQ^2\right)$. Moreover suppose that $Y \mid Q \sim Geometric(Q)$ and $Q \sim Uniform(0.5,\ 1)$. Calculate

(a) $E(X)$ and (b) $Var(X)$. [**Hint:** You can use that $E(Y \mid Q) = 1/Q$ and $Var(Y \mid Q) = (1-Q)/Q^2$ **].**

Problem 5:

(a) Let X be a random variable with distribution function F. Show that F is right continuous.

(b) Show that $1 - P[\limsup A_n^c] = P(\liminf A_n)$

(c) Suppose that $P(A_n) = 1$, for all n. Show that $P(\cap_{n=1}^{\infty} A_n) = 1$

Problem 6: Let $X_1, X_2, ...$ be a sequence of independent random variables with common distribution function F, mean μ and variance σ^2. Let

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i, \qquad S = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2}$$

(a) Show that $\bar{X} \to_p \mu$ and $S \to_p \sigma$. Carefully (but concisely) justify your claims.

(b) Find the asymptotic distribution of

$$\frac{\sqrt{n}\left(\bar{X} - \mu\right)}{S}$$

(c) Find the asymptotic distribution of

$$\sqrt{n}\left(S^2 - \sigma^2\right).$$

(d) Find the asymptotic distribution of

$$\sqrt{n}\left(\frac{S}{\sigma} - 1\right).$$