STAT 530: Partial Identification

Mar. 22, 2010

Deceptively simple problem

$X \sim$ Bernoulli(r) in population.
Infer r from random sample.

Slight twist: X not measured well
(e.g. perhaps some subjects give wrong answer on questionnaire)

Measure X^* instead of X on sampled subjects, where
$SN = Pr(X^* = 1|X = 1)$
$SP = Pr(X^* = 0|X = 0)$

Still looks relatively simple, presuming some info. on misclassification rates

$X^* \sim$ Bernoulli$\{rSN + (1-r)(1-SP)\}$ in population.

Say decide on prior:

$$p(r, SN, SP) \;\; \propto \;\; I_{(0,1)}(r) I_{(a,1)}(SN) I_{(b,1)}(SP)$$

Two issues

- MCMC and parameterization
- Information flow

MCMC

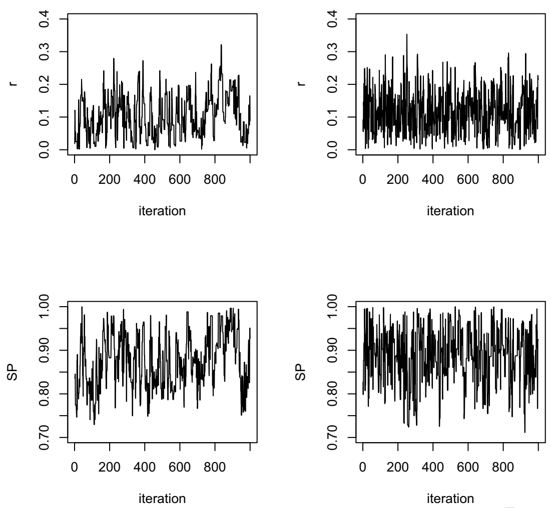
Example

- Data: 20/100 have $X^* = 1$
- Prior information: $a = 0.9$, $b = 0.7$

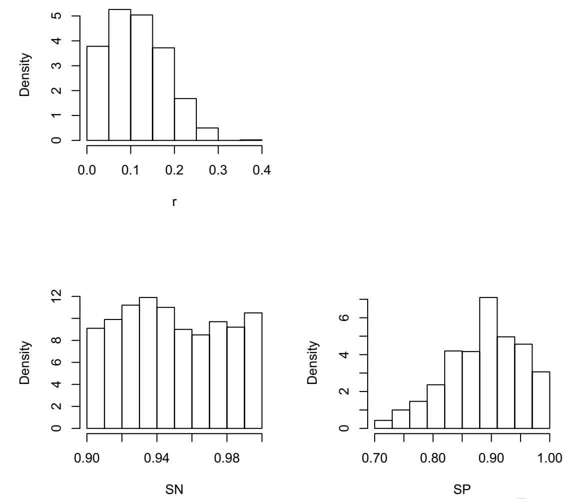
1. Univariate RWMH in original parameterization (r, SN, SP), tuned to have approx. 50% acceptance rate for each component.
2. Univariate RWMH in new parameterization $(\tilde{r}, SN, SP)$, tuned to have approx. 50% acceptance rate for each component.

Intuition for why 2 might work better?

Traceplots



Information flow: Where does the 'extra-prior' info. about SP come from???



Intuition

Whereas r and (SN, SP) are independent *a priori*,
$\tilde{r}$ and (SN, SP) are **dependent** *a priori*

Thus we learn about $\tilde{r}$ directly from the data,
and the prior dependence then implies something about (SN, SP)

[Also bear in mind that the quantity of most interest, r, can be regarded as a function of $(\tilde{r}, SN, SP)$.]

More formally

As $n \to \infty$

$p(\tilde{r}|\text{Data}) \to$

$p(SN, SP|\text{Data}) \to$

We have just seen a simple example of a **partially identified** model

Arise somewhat generally when data are imperfect, but one isn't sure to what extent.

Characterized by the large-sample limit of the posterior distribution on the parameter of interest being:

- narrower than the prior
- wider than a single point

Point estimators (e.g. posterior mean) are necessarily biased.

But interval estimators (e.g. credible interval) reflect this appropriately.