STAT 530: Generalized Linear Mixed Effect Models

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Combine two big ideas

Generalized linear model: natural extension of linear model to non-normal Y variable.

Hierarchical model: group-specific parameters (random effects), described by variance component which controls 'shrinkage.'

General set-up

$$
\begin{aligned}
p(y_j|x_j,\beta_j,\gamma) &= \prod_{i=1}^{n_j} p(y_{ij}|\beta_j^T x_{ij},\gamma) \\
\beta_1,\ldots,\beta_m &\overset{\text{iid}}{\sim} N_p(\theta,\Sigma) \\
p(\gamma,\theta,\Sigma) &= p(\gamma)p(\theta)p(\Sigma)
\end{aligned}
$$

Text Ex.: (Y,X) = (Test Score, SES)

$$
\begin{aligned}
Y_{ij} &\sim N(\beta_{j1}+\beta_{j2}X_{ij},\sigma^2) \\
\begin{pmatrix}\beta_{j1}\\ \beta_{j2}\end{pmatrix} &\sim N_2\left(\begin{pmatrix}\theta_1\\ \theta_2\end{pmatrix},\Sigma\right) \\
p(\sigma^2,\theta,\Sigma) &= p(\sigma^2)p(\theta)p(\Sigma)
\end{aligned}
$$

Text Ex.: (Y,X) = (Tumour count, location)

$$\begin{aligned} Y_{ij} &\sim Poisson\left[\exp\left\{\beta_{j1} + \beta_{j2}(i/20) + \ldots + \beta_{j5}(i/20)^4\right\}\right] \\ \beta_1, \ldots, \beta_m &\overset{\text{iid}}{\sim} N_p(\theta, \Sigma) \\ p(\gamma, \theta, \Sigma) &= p(\gamma)p(\theta)p(\Sigma) \end{aligned}$$

Sometimes 'structured' random effects make sense...

For instance, model 'smooth' dependence of Y on S

$$\begin{aligned} Y_{ij} &\sim ???\left(\sum_{k=1}^{p} \beta_{jk} b_k(S_{ij})\right) \\ \beta_j &\overset{\text{iid}}{\sim} N_p\left((\mu(\theta), \Sigma(\lambda^2, \tau^2)\right) \\ p(\theta, \lambda^2, \tau^2) &= p(\theta)p(\lambda^2)p(\tau^2) \end{aligned}$$

Set up in such a way that τ^2 governs the smoothness of $E(Y|S)$.

Or in a spatial context

If β_{jk} represents effect (in group j) at spatial location k, set up $\beta_j \sim N(\mu(\theta), \Sigma(\lambda^2, \tau^2))$ such that:

- $\Sigma(\lambda^2, 0) = \lambda^2 I$
- whereas for large τ^2, $\Sigma_{kl}(\lambda^2, \tau^2)$ is large if sites k and l are neighbours

Computation for GLMM

1. $(\theta|\beta, \Sigma, y, x)$
2. $(\Sigma|\theta, \beta, y, x)$
3. $(\beta_j|\beta_{-j}, \theta, \Sigma, y, x)$

INLA: a recent computational breakthrough for GLMM?

- Integrated nested Laplace approximation, Rue and Martino (JRSS-B, 2009)
- Combines Lapace approximation and numerical integration ‘in a very efficient manner’
- R package
- Time will tell!