Stepwise Example (low birthweight dataset again)

```
fit0 <- glm(low ~ ., family=binomial, data=bwt)

fit1 <- stepAIC(fit0, ~.)

fit2 <- stepAIC(fit0, ~.^2 + I(scale(age)^2) +
                             I(scale(lwt)^2)   )

fit3 <- stepAIC(fit1, ~.^2 + I(scale(age)^2) +
                             I(scale(lwt)^2)  )
```

```
> summary(fit0)
            Estimate Std. Error z value Pr(>|z|)
(Intercept)  0.82302    1.24471   0.661  0.50848
age         -0.03723    0.03870  -0.962  0.33602
lwt         -0.01565    0.00708  -2.211  0.02705 *
raceblack    1.19241    0.53597   2.225  0.02609 *
raceother    0.74069    0.46174   1.604  0.10869
smokeTRUE    0.75553    0.42502   1.778  0.07546 .
ptdTRUE      1.34376    0.48062   2.796  0.00518 **
htTRUE       1.91317    0.72074   2.654  0.00794 **
uiTRUE       0.68019    0.46434   1.465  0.14296
ftv1        -0.43638    0.47939  -0.910  0.36268
ftv2+        0.17901    0.45638   0.392  0.69488
---
Residual deviance: 195.48  on 178  degrees of freedom
AIC: 217.48
```

```
> fit1$anova
Stepwise Model Path
Analysis of Deviance Table

Initial Model:
low ~ age + lwt + race + smoke + ptd + ht + ui + ftv

Final Model:
low ~ lwt + race + smoke + ptd + ht + ui


   Step Df Deviance Resid. Df Resid. Dev      AIC
1                         178   195.4755 217.4755
2 - ftv  2 1.358185       180   196.8337 214.8337
3 - age  1 1.017866       181   197.8516 213.8516
```

```
> fit2$anova
Stepwise Model Path
Analysis of Deviance Table

Initial Model:
low ~ age + lwt + race + smoke + ptd + ht + ui + ftv

Final Model:
low ~ age + lwt + smoke + ptd + ht + ui + ftv + age:ftv +
                                               smoke:ui

        Step Df   Deviance Resid. Df Resid. Dev      AIC
1                                 178   195.4755 217.4755
2  + age:ftv  2 12.474896        176   183.0006 209.0006
3 + smoke:ui  1   3.056805        175   179.9438 207.9438
4     - race  2   3.129586        177   183.0734 207.0734
```

```
> fit3$anova
Stepwise Model Path
Analysis of Deviance Table

Initial Model:
low ~ lwt + race + smoke + ptd + ht + ui

Final Model:
low ~ lwt + race + smoke + ptd + ht + ui


  Step Df Deviance Resid. Df Resid. Dev      AIC
1                         181   197.8516 213.8516
```

```
> summary(fit2)
                  Estimate Std. Error z value Pr(>|z|)
(Intercept)      -0.582374   1.421613  -0.410 0.682057
age               0.075539   0.053967   1.400 0.161599
lwt              -0.020373   0.007497  -2.717 0.006580 **
smokeTRUE         0.780044   0.420385   1.856 0.063518 .
ptdTRUE           1.560317   0.497001   3.139 0.001693 **
htTRUE            2.065696   0.748743   2.759 0.005800 **
uiTRUE            1.818530   0.667555   2.724 0.006446 **
ftv1              2.921088   2.285774   1.278 0.201270
ftv2+             9.244907   2.661497   3.474 0.000514 ***
age:ftv1         -0.161824   0.096819  -1.671 0.094642 .
age:ftv2+        -0.411033   0.119144  -3.450 0.000561 ***
smokeTRUE:uiTRUE -1.916675   0.973097  -1.970 0.048877 *
Residual deviance: 183.07  on 177  degrees of freedom
AIC: 207.07
```

Now - try the same stepwise procedures using BIC.

```
> fit1a <- stepAIC(fit0, ~., k=log(nrow(bwt)))
> fit1a$anova
Initial Model:
low ~ age + lwt + race + smoke + ptd + ht + ui + ftv
Final Model:
low ~ lwt + ptd + ht

     Step Df Deviance Resid. Df Resid. Dev      AIC
1                          178   195.4755 253.1347
2    - ftv  2 1.358185       180   196.8337 244.0094
3    - age  1 1.017866       181   197.8516 239.7855
4   - race  2 7.614209       183   205.4658 236.9163
5 - smoke  1 2.046576       184   207.5124 233.7211
6     - ui  1 2.611024       185   210.1234 231.0904
```

```
> fit2a <- stepAIC(fit0, ~.^2 +  I(scale(age)^2) +
                  I(scale(lwt)^2), k=log(nrow(bwt)))
> fit2a$anova
Initial Model:
low ~ age + lwt + race + smoke + ptd + ht + ui + ftv
Final Model:
low ~ lwt + ptd + ht

     Step Df Deviance Resid. Df Resid. Dev      AIC
1                           178   195.4755 253.1347
2   - ftv  2 1.358185       180   196.8337 244.0094
3   - age  1 1.017866       181   197.8516 239.7855
4  - race  2 7.614209       183   205.4658 236.9163
5 - smoke  1 2.046576       184   207.5124 233.7211
6    - ui  1 2.611024       185   210.1234 231.0904
```

More purely empirical model comparison?

CROSS-VALIDATION

- Randomly split data intro *training* (T) and *validation* (V) cases.
- Fit model to (X_T, Y_T) data.
- Use the fitted model to generate predictions Y_V^* given X_V.

How close is Y_V^* to the actual Y_V?

One formalization - pick the model M for which

$$\log f_M(y_V|x_V, \hat{\theta}_T)$$

is largest.

Biggest model doesn't necessarily win.

Sensitivity to random split?

k-fold cross-validation

Randomly split cases into k blocks: (Y_j, X_j), $j = 1, \ldots, k$.

Let $(Y_{(j)}, X_{(j)})$ denote all data except (Y_j, X_j).

Do cross-validation k times, each time with $k-1$ blocks as training data, one block as validation data.

Aggregate results. For instance choose model for which

$$\sum_{j=1}^{k} \log f_M(y_j | x_j, \hat{\theta}_{(j)})$$

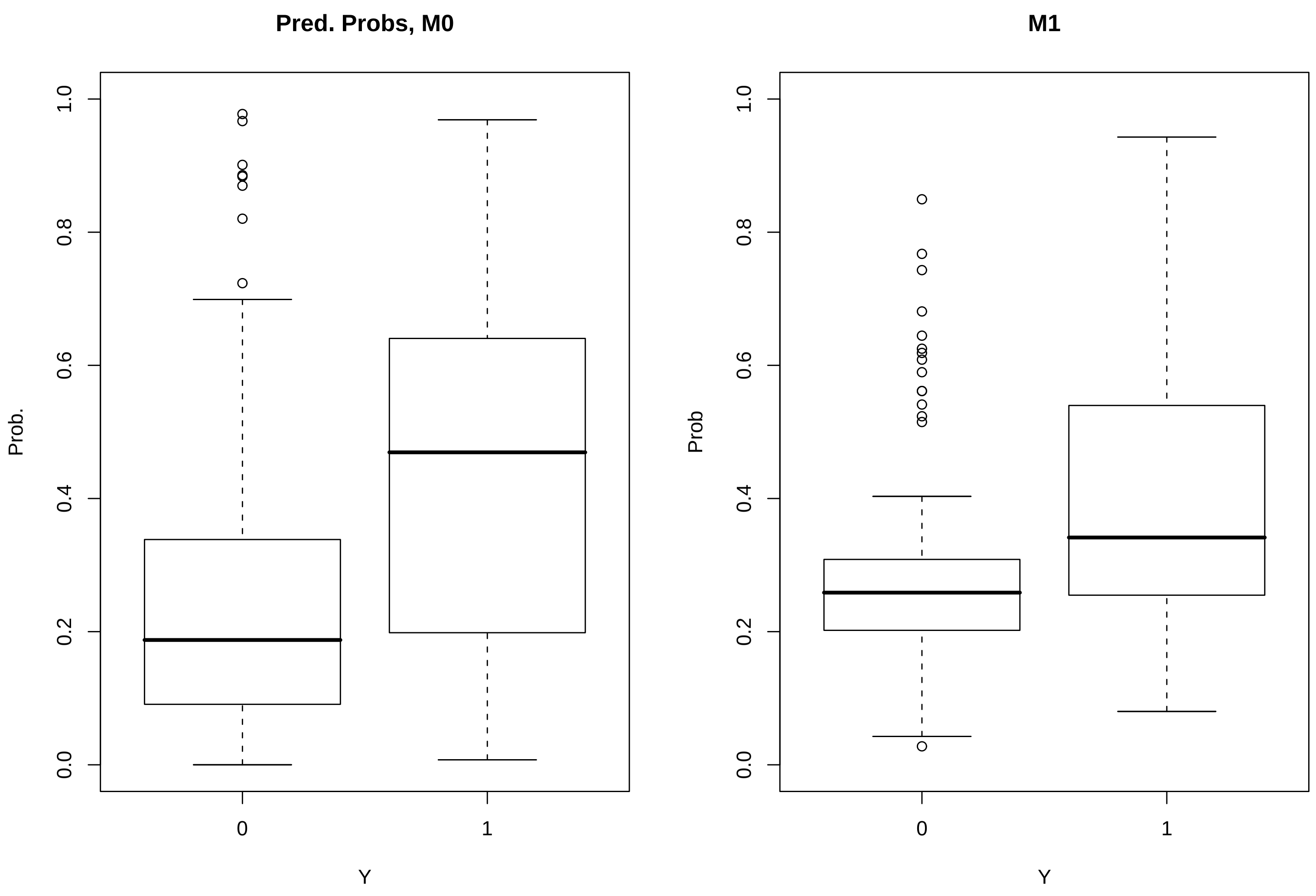
is largest.

Ex.: Compare our AIC and BIC champions.

```
### randomly assign 189 subjects to five blocks
ind <- sample( c(rep(1,38), rep(2,38), rep(3,38),
                 rep(4,38), rep(5,37)) )

for (i in 1:5) {
### fit models to all but i-th block
m0 <- glm(low~age+lwt+smoke+ptd+ht+ui+ftv+age:ftv+smoke:ui,
          family=binomial, data=bwt, subset=(ind!=i) )
m1 <- glm(low~lwt+ptd+ht,
          family=binomial, data=bwt, subset=(ind!=i) )

### predicted prob(Y=1) for i-th block
ftpr0[ind==i] <- predict(m0, newdata=bwt,
                         type="response")[ind==i]
ftpr1[ind==i] <- predict(m1, ...)
```

```
### predictive log-likelihoods and magnitude of diff.
> predll0 <- sum(as.numeric(bwt$low)*log(ftpr0) +
                (1-as.numeric(bwt$low))*log(1-ftpr0))
> predll1 <- sum(as.numeric(bwt$low)*log(ftpr1) +
                (1-as.numeric(bwt$low))*log(1-ftpr1))

> c(predll0, predll1)
 -341.5994 -274.8250

> exp((predll0-predll1)/189)
 0.7023638
```

HUGE preference for second (smaller, BIC-champ.) model. Why?


Pred. Probs, M0
Prob.
0.0
0.2
0.4
0.6
0.8
1.0
0
1
Y
M1
Prob
0.0
0.2
0.4
0.6
0.8
1.0
0
1
Y