

## Web appendix

Supplemental figures for manuscript “Impact of Outcome Model Misspecification on Regression and Doubly-robust Inverse Probability Weighting to Estimate Causal Effect” by G. Lefebvre and P. Gustafson.

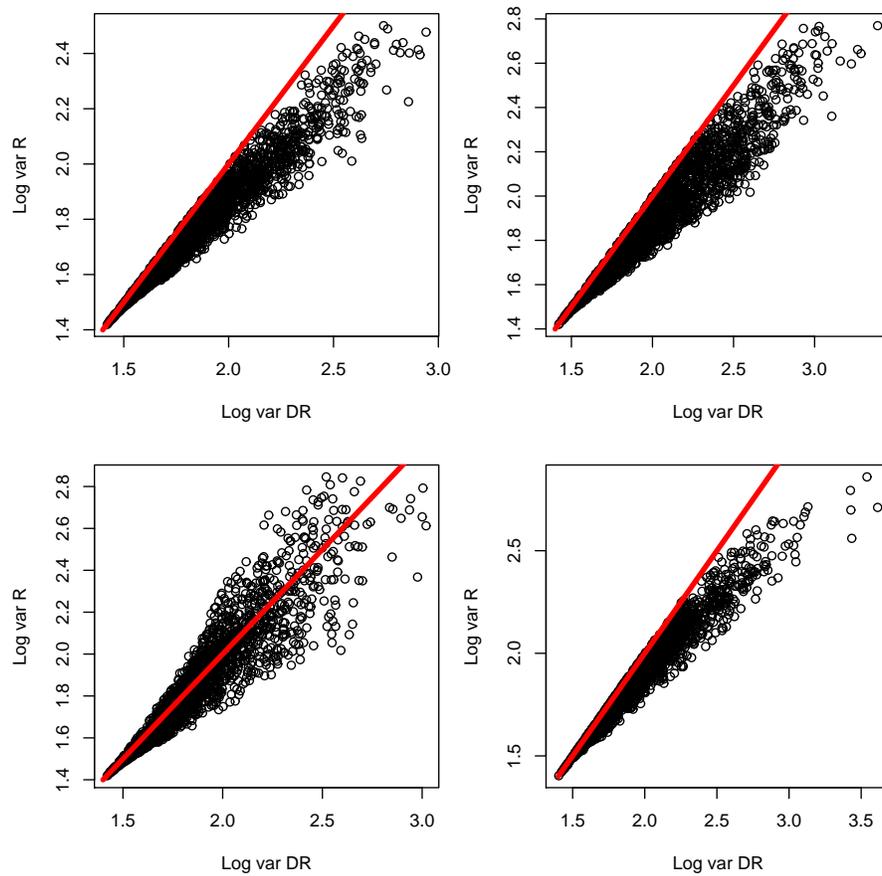


Figure 7: Variance of the regression and doubly robust estimators for Scenarios 1, 4, 5 and 7 ( $v_R^*$  vs  $v_{DR}^*$  on log scale). Display: left to right and top to bottom, respectively.

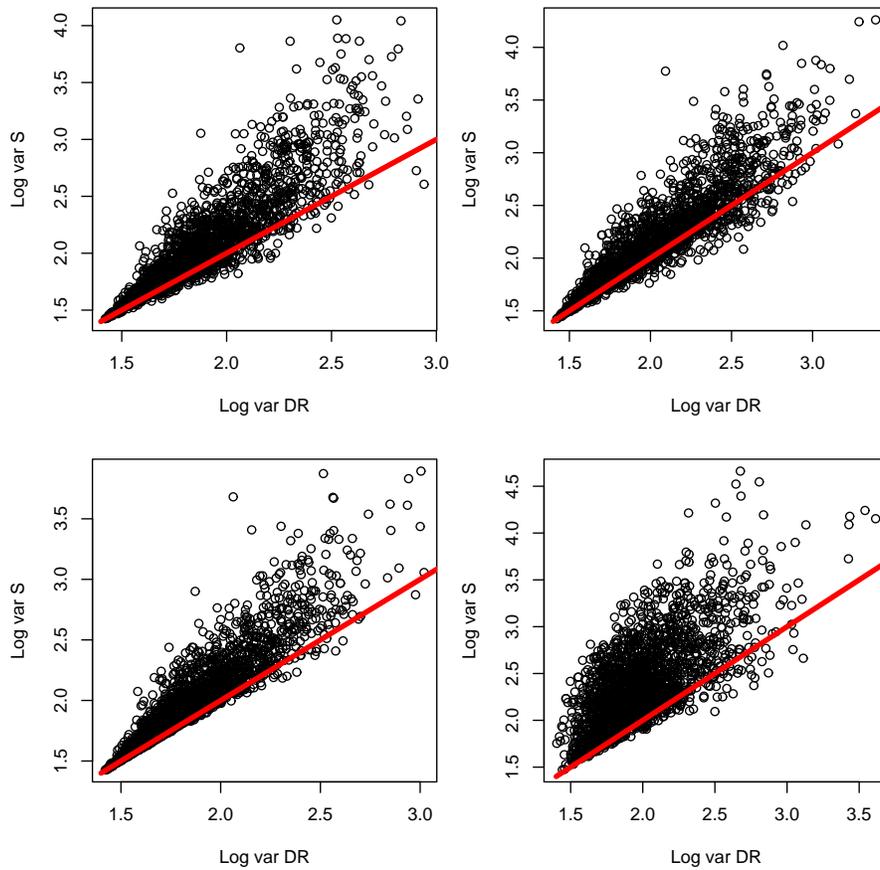


Figure 8: Variance of the stabilized and doubly robust estimators for Scenarios 1, 4, 5 and 7 ( $v_S$  vs  $v_{DR}^*$  on log scale). Display: left to right and top to bottom, respectively.

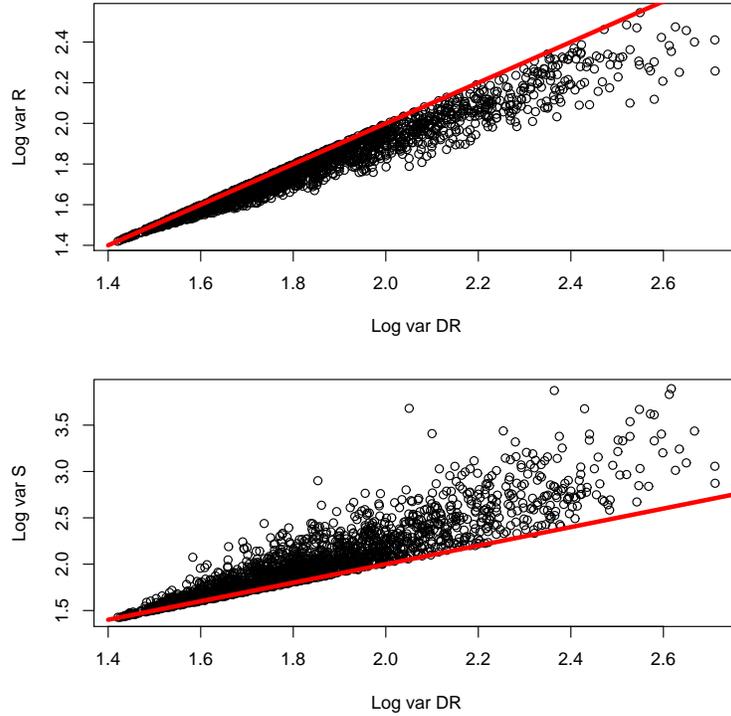


Figure 9: Scenario 5. Top: Variance of the correct regression and doubly robust estimators ( $v_R$  vs  $v_{DR}$  on log scale). Bottom: Variance of the correct stabilized and doubly robust estimators ( $v_S$  vs  $v_{DR}$  on log scale).

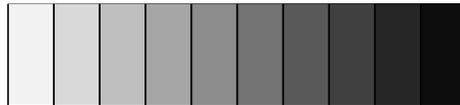


Figure 10: Grey level scale from 0.05 to 0.95 by 0.10 (0: white - 1: black).

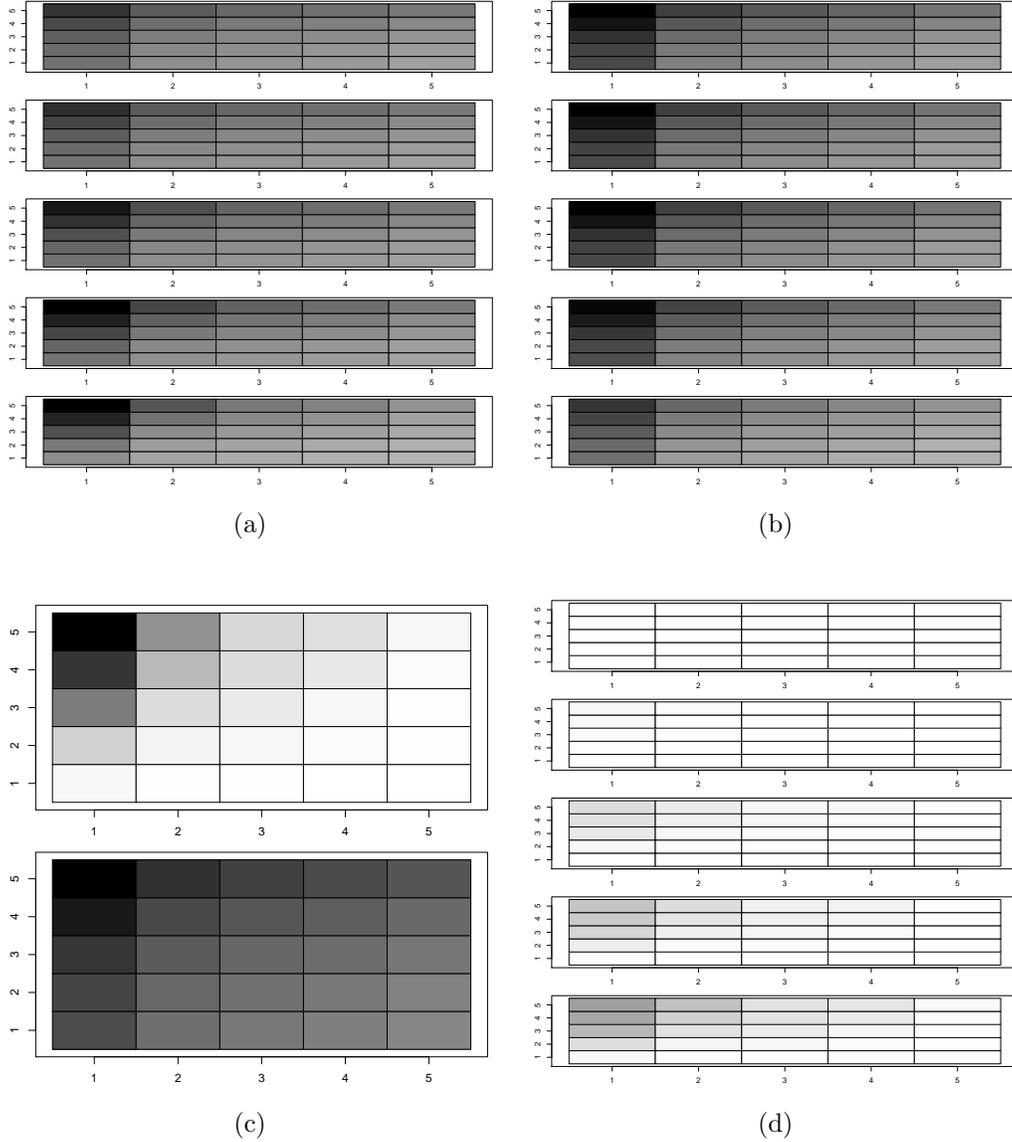


Figure 11: Scenario 1 ( $C_1C_2$  misspecification): Intensity plots as a function of 0-20, 20-40, 40-60, 60-80, and 80-100th percentiles of  $E[e(C)(1 - e(C))]$  (x-axis) and of  $|\beta_m|$  (y-axis) (based on 2000 different  $(\alpha, \beta)$ ). (a) Top to bottom: Average  $MSE_R^*(n)$ ,  $n = 1, 100, 500, 1000, 2000$  (b) Top to bottom: Average  $MSE_{DR}^*(n)$ ,  $n = 1, 100, 500, 1000, 2000$  (c) Top to bottom: Average  $b_R^2$ ,  $v_R^*$ . (d) Top to bottom: Average  $b_R^2/MSE_R^*(n)$ ,  $n = 1, 100, 500, 1000, 2000$ . **Notes:** for better comparisons between  $\hat{\Delta}_R$  and  $\hat{\Delta}_{DR}$ , the grey levels of plots appearing in subfigures (a) and (b) are normalized using a common scale (for each sample size separately). Plots in (c) are drawn on their own individual scale. Plots in (d) are left unnormalized since  $b_R^2/MSE_R^*(n) \in [0, 1]$ . See Figure 10 for grey level scale.

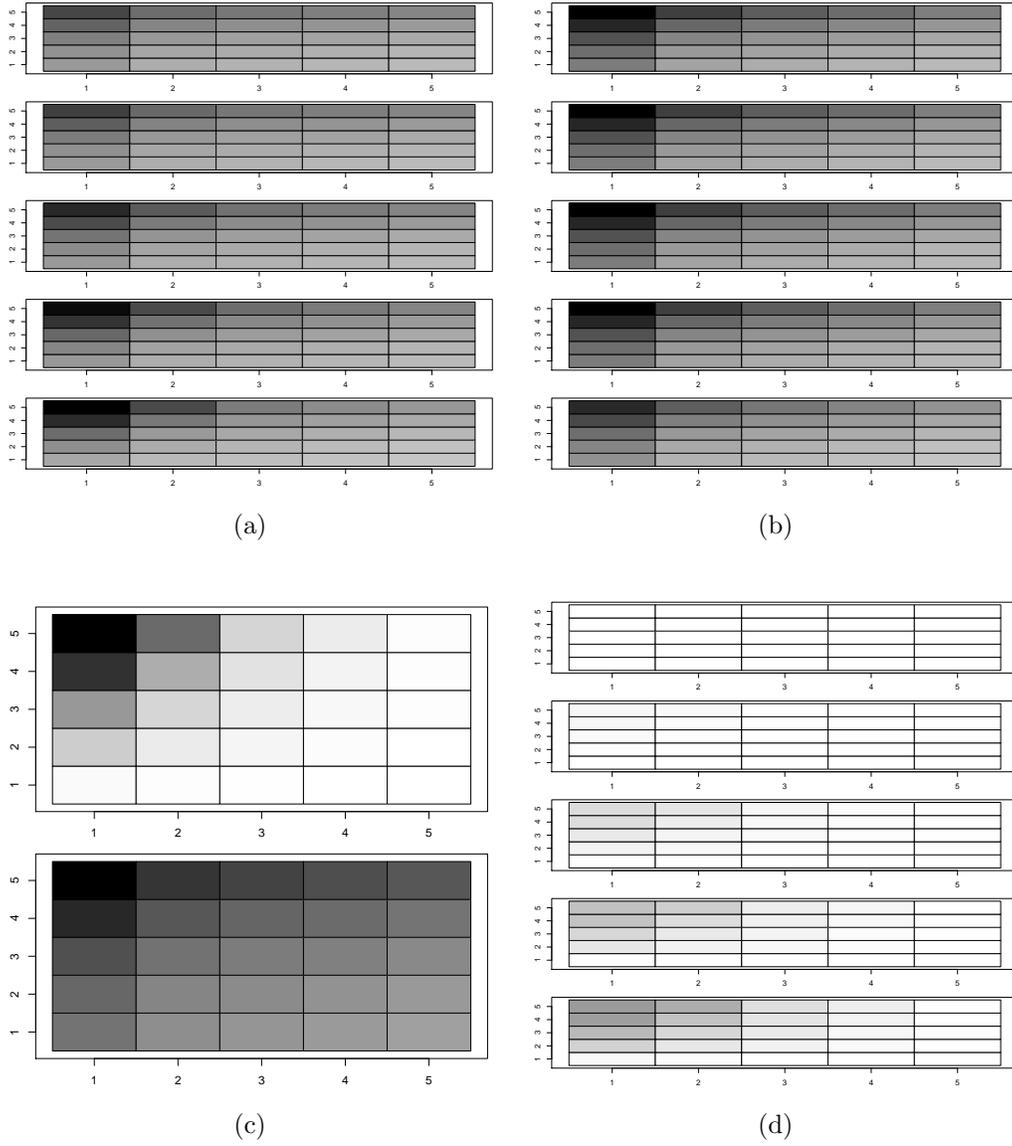


Figure 12: Scenario 4 ( $C_2^2$  misspecification). Layout and legend as per Figure 11.

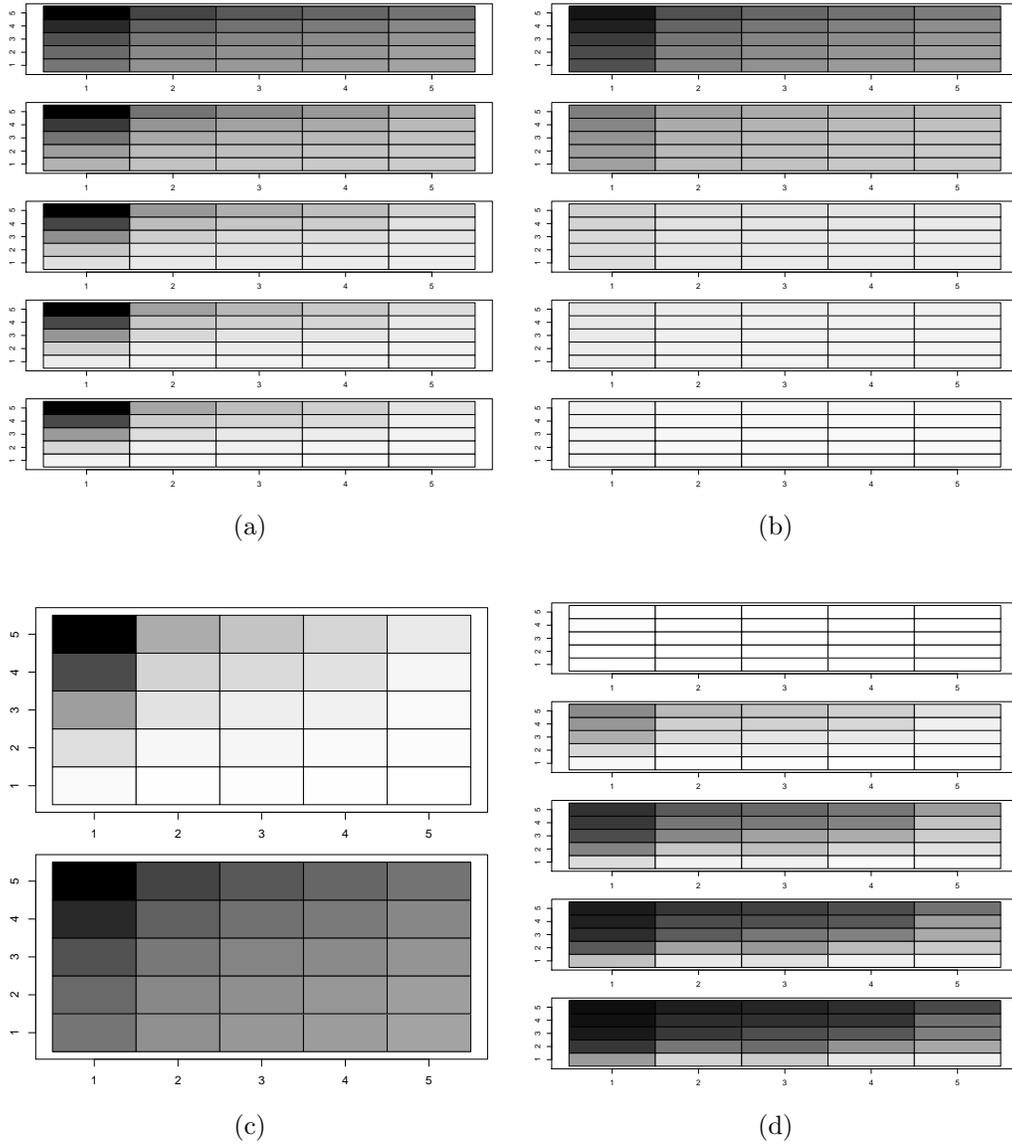


Figure 13: Scenario 5 ( $XC_1$  misspecification). Layout and legend as per Figure 11.

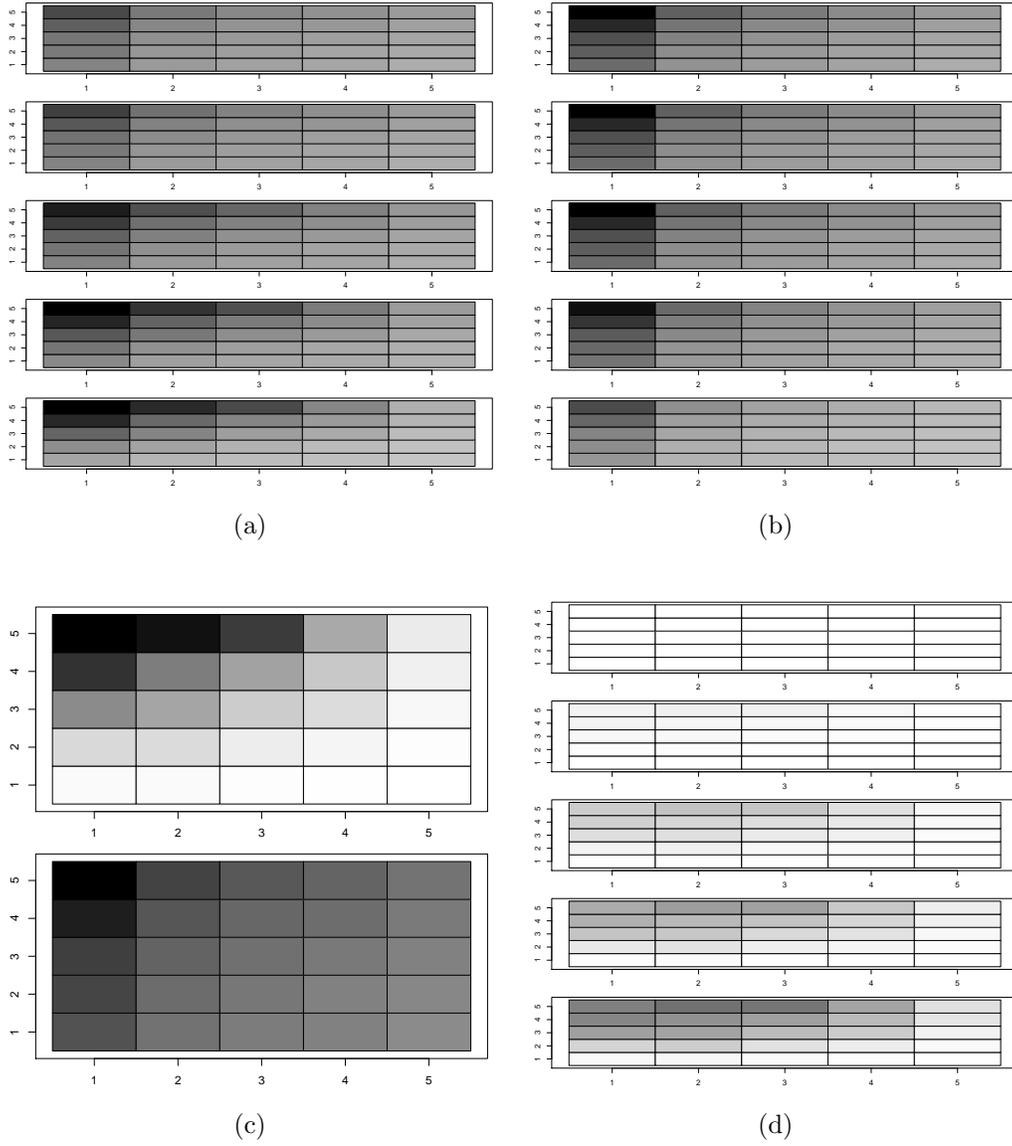
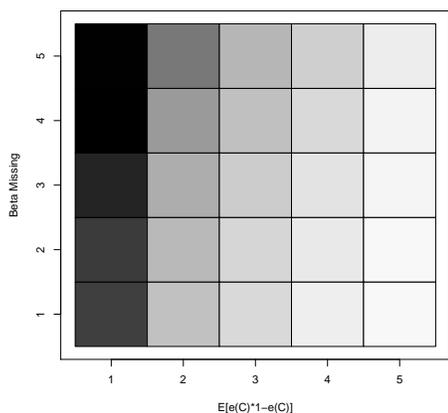
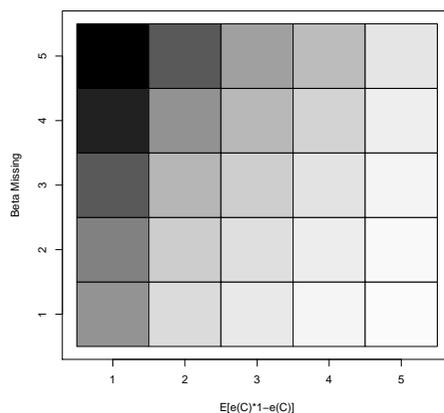


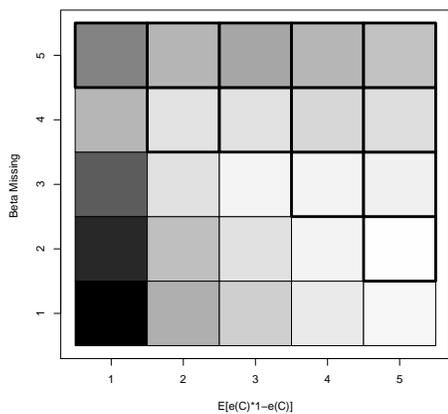
Figure 14: Scenario 7 ( $XC_2^2$  misspecification). Layout and legend as per Figure 11.



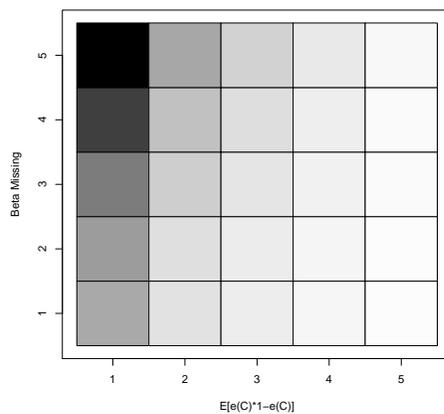
(a) Scenario 1



(b) Scenario 4



(c) Scenario 5



(d) Scenario 7

Figure 15: Intensity plots for the difference in variance between the doubly robust and regression estimators ( $v_{DR}^* - v_R^*$ ) for Scenarios 1, 4, 5 and 7 as a function of 0-20, 20-40, 40-60, 60-80, and 80-100th percentiles of  $E[e(C)(1 - e(C))]$  (x-axis) and of  $|\beta_m|$  (y-axis) (based on 2000 different  $(\alpha, \beta)$ ). **Note:** Bold borders indicate where  $v_{DR}^* - v_R^* < 0$ .

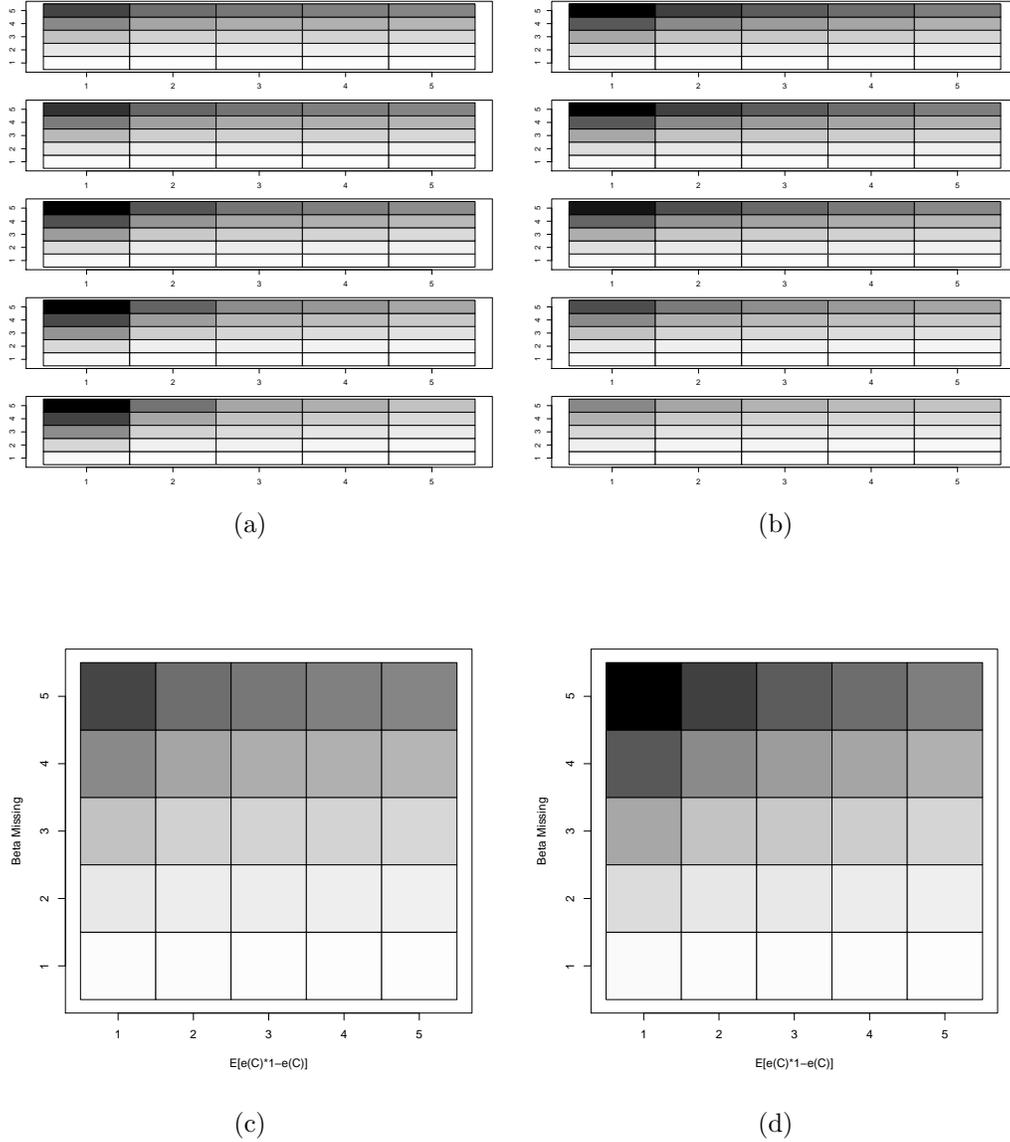
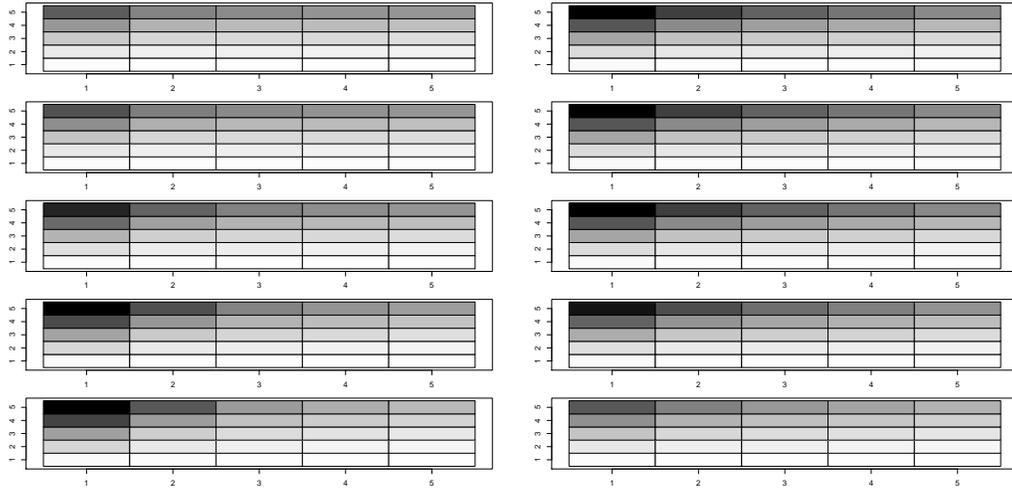
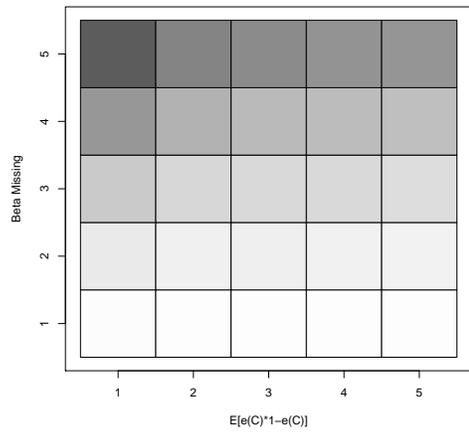


Figure 16: Scenario 1 ( $C_1C_2$  misspecification): Intensity plots as a function of 0-20, 20-40, 40-60, 60-80, and 80-100th percentiles of  $E[e(C)(1 - e(C))]$  (x-axis) and of  $|\beta_m|$  (y-axis) (based on 2000 different  $(\alpha, \beta)$ ). (a) Top to bottom: Average  $MSE_R^*(n) - MSE_R(n)$ ,  $n = 1, 100, 500, 1000, 2000$  (b) Top to bottom: Average  $MSE_{DR}^*(n) - MSE_{DR}(n)$ ,  $n = 1, 100, 500, 1000, 2000$  (c) Average  $v_R^* - v_R$  (d) Average  $v_{DR}^* - v_{DR}$ . **Note:** for better comparisons between  $\hat{\Delta}_R$  and  $\hat{\Delta}_{DR}$ , the grey levels of plots appearing within subfigure pairs (a,b) and (c,d) are normalized using a common scale (for each sample size separately when applicable).

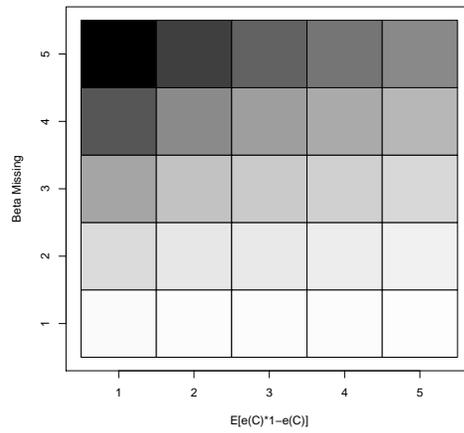


(a)

(b)



(c)



(d)

Figure 17: Scenario 4 ( $C_2^2$  misspecification). Layout and legend as per Figure 16.

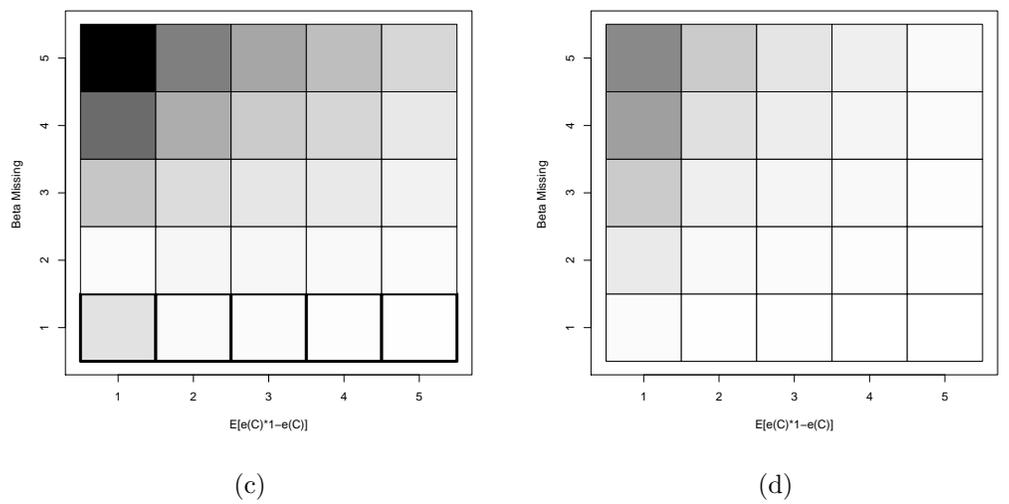
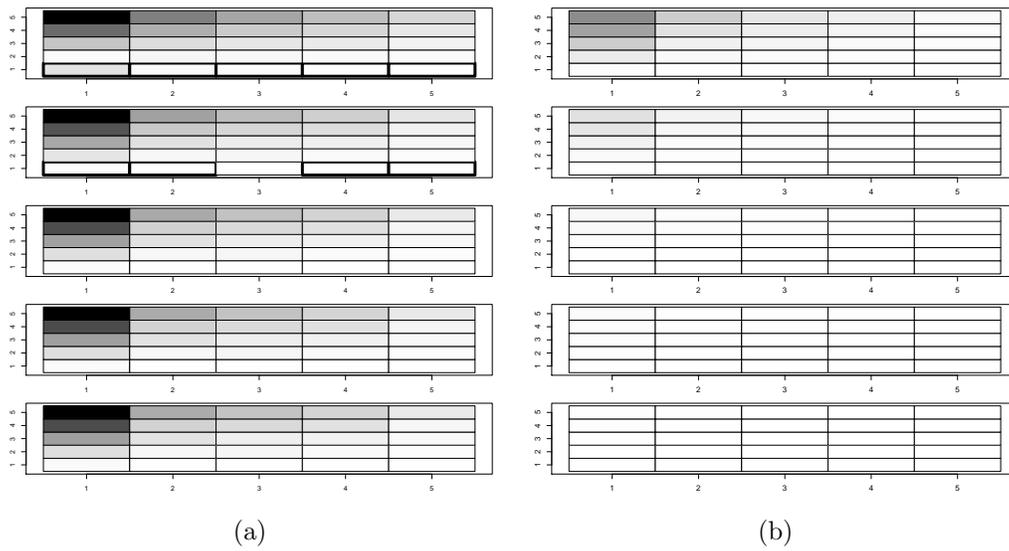
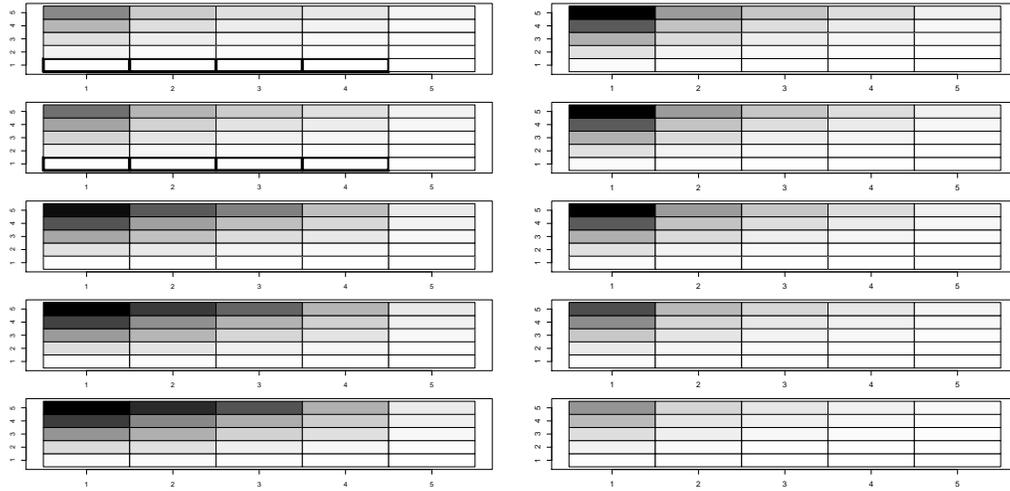
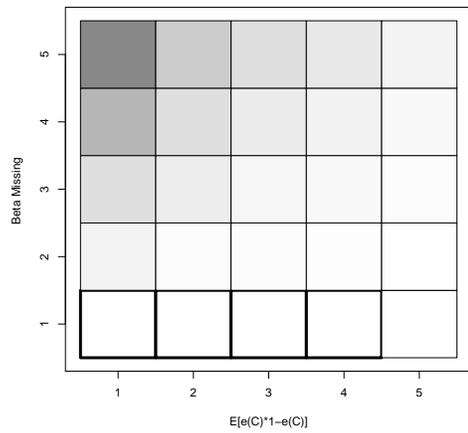


Figure 18: Scenario 5 ( $XC_1$  misspecification): Layout and legend as per Figure 16. Bold borders indicate negative value for the quantity of interest.

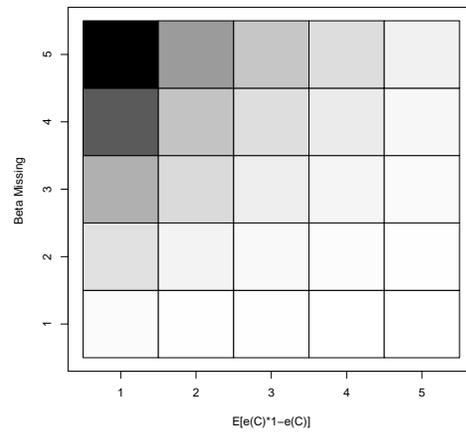


(a)

(b)



(c)



(d)

Figure 19: Scenario 7 ( $XC_2^2$  misspecification). Layout and legend as per Figure 18.