# STAT 460/560- Assignments

Week II: Due: Sept 27 in class.

1. Follow Example 3.1 in the Lecture Notes, show that both Negative Binomial and Poisson distributions are one-parameter exponential families.

2. Follow Example 3.3 in the Lecture Notes,

   (a) Show that two-parameter Gamma distribution family is a multiple parameter exponential family, and select a $\boldsymbol{T}$ so that $\text{VAR}(\boldsymbol{T}; \eta)$ is positive definite.

   (b) Show that multinomial distribution family with fixed number of trials $n$ is a multiple parameter exponential family, and select a $\boldsymbol{T}$ such that $\text{VAR}(\boldsymbol{T}; \eta)$ is positive definite.

3. Given Binomial distribution Binomial$(n, p)$ and the exponential distribution $\exp(\theta)$ families,

   (a) Compute the score function and the Fisher Information of these two distributions.

   (b) Verify that the score functions of these two distributions have expectation 0.

   (c) Verify that the sample means of these two distributions attains the lower bound derived from the Cramér-Rao Information Inequality (Theorem 4.1, Lecture Notes).

4. Let $X_1, X_2, \ldots, X_n$ be an i.i.d. random sample from $N(\mu, \sigma^2)$. Let $\bar{X}_n = (1/n)\sum_{i=1}^n X_i$ be the sample mean and $S_n^2 = \frac{1}{n-1}\sum_{i=1}^n (X_i - \bar{X}_n)^2$ be the sample variance.

   (a) Find the bias of the sample standard deviation $S_n$ as an estimator of the population standard error $(\sigma)$. (Hint: use the fact that $S_n^2$ has a distribution related to $\chi^2_{n-1}$.)

   (b) Find the MSE of the sample standard deviation $S_n$.

5. Find the information lower bound for estimating $\eta = \exp(-\theta)$ under Poisson distribution model whose mean is $\theta$ and derive the UMVUE for $\eta$ given a random sample of $n$ i.i.d. observations.

   **Remark: we refer information lower bound as** $\mathbb{I}^{-1}(\eta)$.