Solutions to Some Problems from Chapter 6

6.3. This sample is from a standard normal distribution, so we know that $\bar{X} \sim \mathcal{N}(0, \frac{1}{16})$, an exact result (no approximation!). Hence

$$\frac{\bar{X}}{\sqrt{1/16}} = 4\bar{X} \sim \mathcal{N}(0,1),$$

and

$$P(|\bar{X}| < c) = 0.5 \quad \Leftrightarrow \quad P(|4\bar{X}| < 4c) = 0.5 \quad \Leftrightarrow \quad P(-4c < Z < 4c) = 0.5.$$

Equivalently (draw pictures if you don't see it immediately!)

$$P(0 < Z < 4c) = 0.25 \quad \Leftrightarrow \quad P(-\infty < Z < 4c) = 0.75.$$

From the table, this requires $4c \approx 0.675$, or $c \approx 0.169$.

6.4. We are told that $T \sim t_7$. By similar reasoning to above

$$P(|T| < t_0) = 0.9 \quad \Leftrightarrow \quad P(-t_0 < T < t_0) = 0.9 \quad \Leftrightarrow \quad P(-\infty < T < t_0) = 0.95.$$

From the table, with $df = 7$, this requires $t_0 = 1.895$.

For $P(T > t_0) = 0.05 \Leftrightarrow P(T < t_0) = 0.95$, we also require $t_0 = 1.895$.

6.8. By change-of-variables (Proposition B on p.62), or using moment generating functions, you can easily show that if $X \sim \text{Gamma}(\alpha,\ \lambda)$ and $a > 0$, then

$$aX \sim \text{Gamma}(\alpha,\ \lambda/a).$$

Hence, if $X \sim \exp(1) = \text{Gamma}(1,1)$, then $2X \sim \text{Gamma}(1, \frac{1}{2}) = \text{Gamma}(\frac{2}{2}, \frac{1}{2}) = \chi^2_2$. By the definition of F distribution, because X and Y are independent, we have

$$\frac{X}{Y} = \frac{2X}{2Y} = \frac{2X/2}{2Y/2} \sim F_{2,2}.$$

For the $\exp(\lambda) = \text{Gamma}(1,\ \lambda)$ case, we have $\lambda X \sim \text{Gamma}(1,\ 1)$. So,

$$\frac{X}{Y} = \frac{\lambda X}{\lambda Y} \sim F_{2,2}$$

by the same reasoning as above.