FINAL EXAMINATION

Statistics 305

Term 2, 2006-2007

Wednesday, April 18, 2007

Time: 3:30pm – 6:00pm

Student Name (**Please print in caps**): ______________________________

Student Number: ______________________________

Notes:

- This exam has 8 problems on the 10 following pages, plus 3 pages of statistical tables. Check to ensure that you have a complete paper.
- The amount each part of each question is worth is shown in [ ] on the left-hand side of the page.
- Where appropriate, record your answers in the blanks provided on the right-hand side of the page.
- Your solutions **must be justified**; show all the work and **state all the reason(s) leading to your answer** for each question in the space provided immediately under the question.
- Clear and complete solutions are essential; little partial credit will be given.
- This is a closed book exam.
- A **single** two-sided 8.5 x 11 page of notes is allowed.
- Calculators are allowed (but not for symbolic differentiation or integration).
- No devices (including calculators) that can store text or send/receive messages are allowed.

Problem	Total Available	Score
1.	11	
2.	18	
3.	11	
4.	13	
5.	7	
6.	12	
7.	18	
8.	10	
Total	100	

1. Suppose the random variables X and Y have:

$$E(X) = 2, \qquad E(Y) = 3,$$
$$SD(X) = 1, \qquad SD(Y) = 4,$$
$$\text{and } Corr(X,Y) = 0.5.$$

[2] a) $\mathrm{E}(5X - 3Y + 4) =$ ____________________

[5] b) $\mathrm{SD}(5X - 3Y + 4) =$ ____________________

[4] c) Suppose, in addition, that X and Y are jointly bivariate normally distributed. Evaluate $\Pr(5X - 3Y + 4 > 9) =$ ____________________

2. Suppose X, the lifetime for an electronic device, is an exponential random variable with a mean of 2 years; that is, X has the density function:

$$f(x) = \frac{1}{2}\exp\left(-\frac{x}{2}\right) \qquad \text{for } x > 0.$$

If 100 of these electronic devices are operating independently, approximate the probability that:

[9] a) The average value of the 100 lifetimes is at least 2.3 years. ____________________

[9] b) At least 50 of these 100 devices are still operating after 2 years. ____________________

3. Suppose $\overline{X}$ and S^2 are the sample mean and sample variance of a simple random sample of size $n = 25$ from a normal population with mean $\mu = 3$ and variance $\sigma^2 = 100$.

[5] a) $P(51.7 \le S^2 \le 138.3) =$ ____________________

[6] b) $P(2 \le \overline{X} \le 5 \text{ and } 51.7 \le S^2 \le 138.3) =$ ____________________

4. Suppose $X_1, X_2, \ldots, X_n$ is a simple random sample from a Poisson distribution with parameter λ. Suppose that the target of inference is not λ but the new parameter $\theta = \exp(-\lambda) = P(X = 0)$. Note that $\lambda > 0$, so $0 < \theta < 1$.

Express your answers to b) – d) below entirely in terms of θ (not λ).

[2] a) Find $\hat{\theta}_{MM}$, the method of moments estimator (MME) of θ. __________________

[4] b) Find the asymptotic variance of the MME, $\hat{\theta}_{MM}$. __________________

[5] c) Find a second order approximation to the bias of the MME, $\hat{\theta}_{MM}$.
Is $\hat{\theta}_{MM}$ asymptotically unbiased? __________________

[2] d) What is the asymptotic distribution of the MME, $\hat{\theta}_{MM}$? Explain clearly. __________________

5. Suppose $X_1, X_2, \ldots, X_n$ is a simple random sample from a normal distribution with mean μ and variance σ^2, where both parameters are unknown.

[4] a) Derive the form of the **exact** $1-\alpha$ confidence interval for the population mean μ. __________________

[3] b) Suppose a simple random sample of $n = 16$ from this distribution leads to a sample average of $\bar{x} = 10$ and a sample standard deviation of $s = 5$. Evaluate the **exact** 80% confidence interval for the population mean μ. __________________

6. Suppose $X_1, X_2, \ldots, X_n$ is a simple random sample from the distribution:

$$f_\theta(x) = \theta x^{\theta - 1} \qquad \text{for } 0 \le x \le 1.$$

Note that this is a density function provided that $\theta > 0$.

[4] a) Evaluate the Fisher Information in a single observation X. __________________

[4] b) Find $\hat{\theta}_{ML}$, the maximum likelihood estimator (MLE) of θ. __________________

[4] c) What is the form of the approximate 90% confidence interval for θ based on the MLE $\hat{\theta}_{ML}$ for this problem? Indicate clearly how this result is obtained. __________________

7. Suppose $X_1, X_2, \ldots, X_9$ is a random sample of size $n = 9$ from a $N(\mu, 1)$ population. You have decided to test H_0: $\mu = 0$ versus H_1: $\mu > 0$ by rejecting H_0 if $\bar{x} > 1.0$, where $\bar{x}$ is the observed value of the sample mean $\bar{X}$.

[5] a) What is the probability of Type I error of this test? __________________

[5] b) What is its probability of Type II error when the true value of μ is 1.2? __________________

[3] c) Is this test uniformly most powerful among all tests of H_0: $\mu = 0$ versus H_1: $\mu > 0$ having the significance level calculated in a)? Explain clearly. __________________

[5] d) For this test, what is the smallest sample size that would result in a power of at least 0.90 when the true value of μ is 1.2? __________________

8. According to genetic theory, flies of three types resulting from certain cross-breedings should occur with probabilities θ^2, $2\theta(1-\theta)$ and $(1-\theta)^2$ respectively, where $0 < \theta < 1$ is an unknown parameter. An experimenter carried out n = 190 such cross-breedings and observed corresponding counts of the three types of flies of 15, 68 and 107. Use the generalized likelihood ratio test to assess if the data provide convincing evidence against the null hypothesis that the numbers of the three types of flies follow this genetic model.

Note: You do **not** need to derive the form of the generalized likelihood ratio test but you must indicate **very clearly** the reasons for all the steps in your evaluation.

[10] __________________

THE END