MIDTERM EXAMINATION # 1

Statistics 305

Term 1, 2006-2007

Thursday, October 12, 2006

Time: 9:30am – 10:45am

Student Name (**Please print in caps**): ______________________________

Student Number: ______________________________

Notes:

- This midterm has 5 problems on the 7 following pages, plus 3 pages of statistical tables. Check to ensure that you have a complete paper.
- The amount each part of each question is worth is shown in [ ] on the left-hand side of the page.
- Where appropriate, record your answers in the blanks provided on the right-hand side of the page.
- Your solutions must be justified; show all the work and state all the reason(s) leading to your answer for each question in the space provided immediately under the question.
- Clear and complete solutions are essential; little partial credit will be given.
- This is a closed book exam.
- A single one-sided 8.5 x 11 page of notes is allowed.
- Calculators are allowed (but not for symbolic differentiation or integration).
- No devices (including calculators) that can store text or send/receive messages are allowed.

Problem	Total Available	Score
1.	12	
2.	5	
3	8	
4.	11	
5.	14	
Total	50	

1. Suppose $\boldsymbol{X}$ and $\boldsymbol{Y}$ are independent and identically distributed random variables that are uniformly distributed on [0, 1].

[2] a) $\mathrm{E}(2\boldsymbol{X} - 4\boldsymbol{Y} + 5) =$ ____________________

[3] b) $\mathrm{SD}(2\boldsymbol{X} - 4\boldsymbol{Y} + 5) =$ ____________________

[7] c) What is the probability density function of $\boldsymbol{W} = \boldsymbol{Y} - \boldsymbol{X}$? ____________________

2. Suppose that a measurement has mean μ and standard deviation $\sigma = 2$. We will use $\overline{\boldsymbol{X}}$, the average of $\boldsymbol{n}$ such independent measurements, to estimate the value of μ. How large

[5] a value of $\boldsymbol{n}$ is required to be 90% confident that this estimate will be within 0.1 of the true value; that is, $\boldsymbol{P}(\,|\overline{\boldsymbol{X}} - \mu| \le 0.1\,) = 0.90$? ____________________

3. Suppose $\overline{\boldsymbol{X}}$ and $\boldsymbol{S}^2$ are the sample mean and the sample variance of a simple random sample of size $\boldsymbol{n} = 9$ from a normal population with mean $\mu = 2$ and variance $\sigma^2 = 36$.

[3] a) $\boldsymbol{P}(\,0 \le \overline{\boldsymbol{X}} \le 3\,) =$ ____________________

[3] b) $\boldsymbol{P}(15.7 \le \boldsymbol{S}^2 \le 90.4\,) =$ ____________________

[2] c) $\boldsymbol{P}(\,0 \le \overline{\boldsymbol{X}} \le 3$ and $15.7 \le \boldsymbol{S}^2 \le 90.4\,) =$ ____________________

4. [5] a) Suppose $\boldsymbol{X}$ is normally distributed with mean μ and variance σ^2. Show that $\boldsymbol{M_X(t)}$, the moment generating function of $\boldsymbol{X}$, is given by

$$\boldsymbol{M_X(t)} = \exp(\mu\, \boldsymbol{t} + \tfrac{1}{2}\sigma^2 \boldsymbol{t}^2)\,.$$

[3] b) Show that the relationship between the moment generating function of $\overline{\boldsymbol{X}}$, the mean of a simple random sample of size from any population, and the moment generating function of a single random variable, $\boldsymbol{X}$, from that same population is given by:

$$\boldsymbol{M_{\overline{X}}(t)} = [\boldsymbol{M_X(t/n)}]^{\boldsymbol{n}}$$

[3] c) Using the above results, show that the sample mean from a normal population (with mean μ and variance σ^2) is normally distributed with mean μ and variance $\sigma^2/\boldsymbol{n}$.

5. Suppose $\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots, \boldsymbol{X_n}$ is a simple random sample from the distribution:

$$\boldsymbol{f}_\theta(\boldsymbol{x}) = \theta\, \boldsymbol{x}^{\theta-1} \qquad \text{for } 0 \le \boldsymbol{x} \le 1.$$

Note that this is a density function provided that $\theta > 0$.

[4] a) Find $\hat{\theta}_{MM}$, the method of moments estimator (MME) of θ. ____________________

[5] b) Find an approximation to the variance of the MME $\hat{\theta}_{MM}$. ____________________

[5] c) Find a second-order approximation to the bias of the MME $\hat{\theta}_{MM}$. ____________________

THE END