Midterm I

Thursday October 7th, 2004.
12:30 – 13:50.

Authorized material :

- One letter-size cheat sheet.

- One scientific calculator without wireless communication feature.
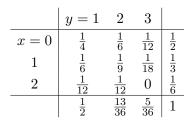
Instructions :

- The exam has 5 pages inlcuding this one.

- Answer all 4 questions; the total number of points is 100.

- Write legibly; give complete solutions.

- You can use the back side of the sheets as drafts. If you use it for writing answers, indicate it clearly.

Last Name :   _____

First Name :   _____

Student Number :   _____

Signature :   _____

Question 1

[25 pts] Are each of the following statements true or false. Justify your answers.

(a) The bivariate mass function of the discrete variables X and Y is given in the following table; X and Y are independent.

	$y = 1$	2	3	
$x = 0$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{2}$
1	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{18}$	$\frac{1}{3}$
2	$\frac{1}{12}$	$\frac{1}{12}$	0	$\frac{1}{6}$
	$\frac{1}{2}$	$\frac{13}{36}$	$\frac{5}{36}$	1

(b) Consider two disjoint events A and B such that $P(A) > 0$ and $P(B) > 0$. Then A and B are independent.

(c) Consider events A and B such that $A \subset B$ and $P(B^C) > 0$. Then $P(A|B) = P(A)$.

(d) The following function is a density function

$$f(x) = \begin{cases} 6x^2 - 1 & \text{if } x \in [0,1] \\ 0 & \text{otherwise} \end{cases}.$$

(e) Consider $X \sim unif(0, Y)$ with $Y \sim exp(\lambda)$, then $E(X) = 1/2\lambda$.

Question 2

(a) [15 pts] Consider the three independent variables X, Y and Z, with variance σ_X^2, σ_Y^2 and σ_Z^2 respectively. Evaluate $cor(U, V)$ where $U = Z + X$ and $V = Z - Y$.

(b) [10 pts] Use the moment generating function to show that if X follows a gamma distribution with parameters α and λ, then $Y = \beta X$, with $\beta > 0$ follows a gamma with parameters α and λ/β.

Question 3

[25 pts] The annual number of hurricanes striking an island in the Atlantic ocean follows a Poisson distribution with parameter λ. Every time a hurricane strikes, the hospital on the island has probability p of being damaged. Whether the hospital is damaged or not is independent from a storm to another. What is the probability that the hospital will not suffer hurricane damage next year.

Question 4

[25 pts] Consider X and Y two independent exponential random variables having the same expectation. Find the distribution of

$$U = \frac{X}{X+Y} \ .$$