MIDTERM EXAMINATION # 1

Statistics 305

Term 1, 2005-2006

Thursday, October 13, 2005

Time: 9:30am – 10:45am

Student Name (**Please print in caps**): ______________________________

Student Number: ______________________________

Notes:

- This midterm has 6 problems on the 6 following pages, plus a final page containing a table of the standard normal distribution. Check to ensure that you have a complete paper.
- The amount each part of each question is worth is shown in [ ] on the left-hand side of the page.
- Where appropriate, record your answers in the blanks provided on the right-hand side of the page.
- Your solutions must be justified; show the work and state the reason(s) leading to your answer for each question in the space provided immediately under the question.
- Clear and complete solutions are essential; little partial credit will be given.
- This is a closed book exam.
- A single one-sided 8.5 x 11 page of notes is allowed.
- Calculators are allowed (but not for symbolic differentiation or integration).
- No devices (including calculators) that can store text or send/receive messages are allowed.

Problem	Total Available	Score
1.	7	
2.	7	
3	15	
4.	5	
5.	10	
6.	6	
Total	50	

1. Suppose the moment generating function (mgf) of the random variable X is given by:

$$M_X(t) = [\exp(t) + 2\exp(2t) + 3\exp(3t)] / 6.$$

[2] a) $E(X) =$ __________________

[3] b) $SD(X) =$ __________________

[2] c) What is the distribution of the random variable X? __________________

2. Suppose X is a binomial random variable with parameters n and p.

[2] a) Show that $M_X(t)$, the mgf of X, is given by

$$M_X(t) = [1 - p + p\exp(t)]^n.$$

For what values of t does $M_X(t)$ exist?

[3] b) Show that if $n \to \infty$ and $p \to 0$ in such a way that $np \to \lambda$, then

$$M_X(t) \to \exp\{\lambda [\exp(t) - 1]\}.$$

[2] c) Explain the practical importance of the result in b).

3. Suppose X, the lifetime for an electronic device, is an exponential random variable with a mean of 2 years; that is, $f_X(x) = ½ \exp(-x/2)$ for $x > 0$. Suppose 100 of these electronic devices are operating independently.

Provide an accurate numerical approximation to the probability that:

[6] a) at least 50 of these 100 devices are still operating after 2 years. __________________

[5] b) less than 3 of these 100 devices are still operating after 8 years. __________________

[4] c) the average value of the 100 lifetimes is at least 2.3 years. __________________

4. Suppose X is a positive random variable with mean μ_X and variance σ_X^2. If $Y = \ln(X)$, where ln = $\log_e$, use the delta method to obtain expressions (in terms of μ_X and σ_X) for:

[1] a) a "first-order" approximation to E(Y). ________________

[2] b) a "first-order" approximation to SD(Y). ________________

[2] c) a "second-order" approximation to E(Y). ________________

5. Suppose X and Y are bivariate normally distributed random variables with means μ_X, μ_Y and variances σ_X^2, σ_Y^2, respectively, and correlation ρ. Then, as given in class, $M_{X,Y}(s,t)$, the joint mgf of X and Y evaluated at s and t, is given by:

$$M_{X,Y}(s,t) = \exp\{\, s\,\mu_X + t\,\mu_Y + [\, s^2\,\sigma_X^2 + 2\,st\,\rho\,\sigma_X\,\sigma_Y + t^2\,\sigma_Y^2\,]\,/2\,\}.$$

Let W = a X + b Y + c, where a, b and c are constants.

a) Give the expressions for:

[1] i) E(W) ________________

[2] ii) Var(W) ________________

[5] b) Evaluate the mgf of W. ________________

[2] c) What is the distribution of W? ________________

6. Suppose X and Y are independent normally distributed random variables with:

$$\mu_X = 8,\ \ \sigma_X^2 = 9 \quad \text{and} \quad \mu_Y = 3,\ \ \sigma_Y^2 = 16.$$

[6] Evaluate P($X > Y$). ________________

THE END