MIDTERM EXAMINATION # 2

Statistics 305

Term 1, 2006-2007

Thursday, November 9, 2006

Time: 9:30am – 10:50am

Student Name (**Please print in caps**): ______________________________

Student Number: ______________________________

Notes:

- This midterm has 5 problems on the 6 following pages, plus 3 pages of statistical tables. Check to ensure that you have a complete paper.
- The amount each part of each question is worth is shown in [ ] on the left-hand side of the page.
- Where appropriate, record your answers in the blanks provided on the right-hand side of the page.
- Your solutions must be justified; show all the work and state all the reason(s) leading to your answer for each question in the space provided immediately under the question.
- Clear and complete solutions are essential; little partial credit will be given.
- This is a closed book exam.
- A single one-sided 8.5 x 11 page of notes is allowed.
- Calculators are allowed (but not for symbolic differentiation or integration).
- No devices (including calculators) that can store text or send/receive messages are allowed.

Problem	Total Available	Score
1.	7	
2.	7	
3	9	
4.	21	
5.	6	
Total	50	

1. Suppose $\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots, \boldsymbol{X}_{\boldsymbol{n}}$ is a simple random sample from a normal population with a mean of μ and variance of σ^2, where both parameters are unknown. Derive the forms of the ***exact*** $1-\alpha$ confidence intervals for:

[3] a) the population mean μ. __________________

[4] b) the population standard deviation σ. __________________

2. Suppose a simple random sample of $\boldsymbol{n} = 9$ from a normal population leads to a sample average of $\bar{\boldsymbol{x}} = 22$ and a sample standard deviation of $\boldsymbol{s} = 6$. Evaluate ***exact*** 90% confidence intervals for:

[3] a) the population mean μ. __________________

[4] b) the population standard deviation σ. __________________

3. Suppose $\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots, \boldsymbol{X}_{\boldsymbol{n}}$ is a simple random sample from the distribution:

$$\boldsymbol{f}_\theta(\boldsymbol{x}) = \theta\, \boldsymbol{x}^{\theta - 1} \qquad \text{for } 0 \le \boldsymbol{x} \le 1.$$

Note that this is a density function provided that $\theta > 0$.

[3] a) Evaluate the Fisher Information in a single observation $\boldsymbol{X}$. __________________

[3] b) Find $\hat{\theta}_{\boldsymbol{ML}}$, the maximum likelihood estimator (MLE) of θ. __________________

[3] c) Derive the form of the approximate $1-\alpha$ confidence interval for θ based on the MLE $\hat{\theta}_{\boldsymbol{ML}}$. __________________

4. Suppose $\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots, \boldsymbol{X}_{\boldsymbol{n}}$ is a simple random sample from the Rayleigh distribution:

$$\boldsymbol{f}_\theta(\boldsymbol{x}) = (\boldsymbol{x}/\theta^2)\exp(-\boldsymbol{x}^2/2\theta^2) \qquad \text{for } \boldsymbol{x} \geq 0,$$

where $\theta > 0$.

[4] a) Find $\hat{\theta}_{\boldsymbol{MM}}$, the method of moments estimator (MME) of θ. ____________________

[5] b) Find the exact variance of $\hat{\theta}_{\boldsymbol{MM}}$, the MME of θ. ____________________

[4] c) Find $\hat{\theta}_{\boldsymbol{ML}}$, the maximum likelihood estimator of θ. ____________________

[4] d) Find the asymptotic variance of $\hat{\theta}_{\boldsymbol{ML}}$, the MLE of θ. ____________________

[4] e) Evaluate the asymptotic relative efficiency of the MME relative to the MLE.
What is the practical interpretation of this result? ____________________

5. Suppose $\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots, \boldsymbol{X}_{\boldsymbol{n}}$ is a simple random sample from a normal population with mean μ and (known) variance = 1.

[4] a) Show that $\overline{\boldsymbol{X}}$ is a sufficient statistic for μ. ____________________

[2] b) What is the practical interpretation of this result? ____________________

THE END