STAT 305 - Midterm 2 - Fall 2003

Time: 60 minutes

[20] Problem 1: Two independent measurements, X and Y, are taken of a random quantity μ.

$$E(X) = E(Y) = \mu$$

but σ_x and σ_Y (the corresponding standard deviations) are unequal. The two measurements are combined by means of a weighted average to give

$$Z = \alpha X + (1 - \alpha) Y$$

where α is a scalar (that is, a number) and $0 \leq \alpha \leq 1$.

(a) Show that $E(Y) = \mu$

(b) Find α in terms of σ_x and σ_Y to minimize $Var(Z)$.

(c) Under what circumstances is it better to use the average $(X + Y)/2$ than either X or Y alone?

[40] Problem 2: A drunkard executes a "random walk" in the following way: Each minute he takes a step north or south, with probability 1/2 each, and his successive step directions are independent. His step length is 50 cm.

(a) Use the central limit theorem to approximate the probability distribution of his location after 1 hour.

(b) Where is he most likely to be?

Make now the assumption that the drunkard has some idea of where he wants to go so that he steps north with probability 2/3 and south with probability 1/3.

(c) Use the central limit theorem to approximate the probability distribution of his location after 1 hour.

(d) Where is he most likely to be?

[20] Problem 3: Show that if X and Y are independent exponential random variables with $\lambda = 1$, then

$$Z = \frac{X}{Y}$$

follows an F distribution. Also identify the degrees of freedom.

[20] Problem 4: Let $X_1, X_2, ..., X_n$ be independent normal random variables with mean μ and variance σ^2. Find the mean and the variance of

$$S^2 = \frac{\sum_{i=1}^{n} \left(X_i - \bar{X}\right)^2}{n-1}.$$