MIDTERM EXAMINATION # 2

Statistics 305

Term 1, 2005-2006

Thursday, November 10, 2005

Time: 9:30am – 10:45am

Student Name (**Please print in caps**): ______________________________

Student Number: ______________________________

Notes:

- This midterm has 5 problems on the 6 following pages, plus 3 pages of tables. Check to ensure that you have a complete paper.
- The amount each part of each question is worth is shown in [ ] on the left-hand side of the page.
- Where appropriate, record your answers in the blanks provided on the right-hand side of the page.
- Your solutions must be justified; show the work and state the reason(s) leading to your answer for each question in the space provided immediately under the question.
- Clear and complete solutions are essential; little partial credit will be given.
- This is a closed book exam.
- A single one-sided 8.5 x 11 page of notes is allowed.
- Calculators are allowed (but not for symbolic differentiation or integration).
- No devices (including calculators) that can store text or send/receive messages are allowed.

Problem	Total Available	Score
1.	10	
2.	6	
3	12	
4.	7	
5.	15	
Total	50	

1. Suppose $X_1, X_2, \ldots, X_n$ is a simple random sample from a normal distribution with mean μ and variance σ^2, where both parameters are unknown.

a) Derive the form of the exact 1 - α confidence interval for:

[3] i) the population mean μ. ________________

[3] ii) the population standard deviation σ. ________________

b) Suppose a simple random sample of n = 16 from this distribution leads to a sample average of $\bar{x} = 10$ and a sample standard deviation of s = 5. Evaluate exact 90% confidence intervals for:

[2] i) the population mean μ. ________________

[2] ii) the population standard deviation σ. ________________

2. Suppose $X_1, X_2, \ldots, X_n$ is a simple random sample from the uniform distribution on the interval from 0 to θ; that is, from the population with density function given by:

$$f_\theta(x) = 1/\theta \qquad \text{for } 0 \le x \le \theta.$$

[2] a) Find $\hat{\theta}_{MM}$, the method of moments estimator (MME) of θ for this example. ________________

[3] b) What is $L(\theta)$, the likelihood function, for this example? Provide a clear sketch. ________________

[1] c) Find $\hat{\theta}_{ML}$, the maximum likelihood estimator (MLE) of θ for this example. ________________

3. Suppose $X_1, X_2, \ldots, X_n$ is a simple random sample from the distribution:

$$f_\theta(x) = \theta x^{\theta - 1} \qquad \text{for } 0 \le x \le 1.$$

Note that this is a density function provided that $\theta > 0$.

[3] a) Find $\hat{\theta}_{MM}$, the method of moments estimator (MME) of θ. ________________

[6] b) Find a second-order approximation to the bias of the MME $\hat{\theta}_{MM}$. ________________

[3] c) Find the asymptotic variance of the MME $\hat{\theta}_{MM}$. ________________

4. Suppose X is a normally distributed random variable with mean μ and variance σ^2, where both parameters are unknown. Evaluate the Fisher Information matrix for
[7] the pair of parameters μ and σ^2 based on the single random variable X.

5. According to the Hardy-Weinberg law, if gene frequencies are in equilibrium, the genotypes *AA*, *Aa* and *aa* occur with relative frequencies $(1-\theta)^2$, $2\theta(1-\theta)$ and θ^2, respectively. Plato et al. (1964) published data on haptoglobin type in a random sample of n = 190 individuals with corresponding counts of 10, 68 and 112. Let X_1, X_2 and X_3 denote the random variables leading to these observed counts (Note: $X_1 + X_2 + X_3 = n$).

[6] a) Find an expression for $\hat{\theta}_{ML}$, the maximum likelihood estimator (MLE) of θ, in terms of the random variables X_1, X_2 and X_3. ____________________

[1] b) Evaluate the MLE $\hat{\theta}_{ML}$ for the given data. ____________________

[4] c) Find an expression for the asymptotic variance of $\hat{\theta}_{ML}$ in terms of n and θ. ____________________

[1] d) Evaluate the estimated standard error of the MLE $\hat{\theta}_{ML}$ for the given data. ____________________

[3] e) Find an approximate 99% confidence interval for θ based on the MLE $\hat{\theta}_{ML}$. ____________________

THE END