STAT 305 - Midterm 3 - Fall 2003

Time: 60 minutes

CHOOSE THREE OF THE FOUR PROBLEMS BELOW

ONLY THE BEST THREE PROBLEMS WILL BE CONSIDERED TO CALCULATE THE GRADE

[1] Student Name _____
Student Number _____

[**33**] Problem 1: One of the earliest applications of the Poisson distribution was made by Student (1907) in studying errors made in counting yeast cells or blood corpuscles with a haemacytometer. In this study, yeast cells were killed and mixed with a gelatin; the mixture was then spread on a glass and allowed to cool. Counts were made on 400 squares and the data are summarized in the following table

\# of Cells	Obs. Freq.	Exp. Freq.	Work Space
0	103		
1	143		
2	98		
3	42		
4	8		
5	4		
6	2		
Total	400		

(a) Find the MLE estimate for the parameter λ

(b) Test the goodness of fit of the Poisson distribution to these data using Pearson's chi-square test.

(c) Find an approximate 95% confidence interval for λ.

Problem 1 (continued)

[**33**] Problem 2: The Pareto distribution has been used in economics as a model for a density function with a slowly decaying tail:

$$f(y,\theta) = \theta y^{-(\theta+1)}, \qquad y \geq 1, \quad \theta > 1.$$

Given an iid sample $y_1, y_2, ..., y_n$

(a) Find the MM estimate of θ

(b) Find the MLE estimate of θ

(c) Find the asymptotic variance of the MLE estimate of θ

(d) Construct an approximate 95% confidence interval for θ using the data ($n = 10$)

1.03 2.70 1.98 1.71 1.34 1.75 3.30 3.35 4.18 3.52

Problem 2 (continued)

[**33**] Problem 3: If gene frequencies are in equilibrium, the genotypes AA, Aa and aa occur with probabilities $(1-\theta)^2$, $2\theta(1-\theta)$ and θ^2, respectively. Plato et al. (1964) published the following data on haptoglobin type in a sample of 190 people:

Genotype	AA	Aa	aa
Frequency	10	68	112
Probability	$(1-\theta)^2$	$2\theta(1-\theta)$	θ^2

(a) Find the MLE of θ
(b) Find the asymptotic variance of the MLE
(c) Find an approximate 99% confidence interval for θ.

Problem 3 (continued)

[**33**] Problem 4: Let $X_1, X_2, ..., X_n$ be iid uniform on $[0, \theta]$.

(a) Find the MM estimate of θ and its mean and variance

(b) Find the MLE of θ

(c) Find the probability density function of the MLE, and calculate its mean and variance

(d) Find a modification of the MLE that renders it unbiased.

Problem 4 (continued)