QUIZ # 1

Statistics 305

Term 2, 2006-2007

Tuesday, January 23, 2007

Time: 2:00pm – 2:30pm

Student Name (**Please print in caps**): SOLUTIONS

Student Number: ______________________

Notes:

- This quiz has 3 problems on the 4 following pages, plus 1 page of statistical tables. Check to ensure that you have a complete paper.
- The amount each part of each question is worth is shown in [ ] on the left-hand side of the page.
- Where appropriate, record your answers in the blanks provided on the right-hand side of the page.
- Your solutions **must be justified**; show **all the work** and state **all the reasons** leading to your answer for each question in the space provided immediately under the question.
- Clear and complete solutions are essential; little partial credit will be given.
- This is a closed book exam.
- A single one-sided 8.5 x 11 page of notes is allowed.
- Calculators are allowed (but not for symbolic differentiation or integration).
- No devices (including calculators) that can store text or send/receive messages are allowed.

Problem	Total Available	Score
1.	7	
2.	7	
3.	11	
Total	25	

1. Suppose the random variables X and Y have:

$E(X) = 1,\quad E(Y) = 2,$

$SD(X) = 3,\quad SD(Y) = 4,$

and $Corr(X, Y) = 0.5$.

[2] a) $E(2X - Y + 5) = 2E(X) - E(Y) + 5$ 5

$= 2(1) - 2 + 5$

$= \underline{\underline{5}}$

[5] b) $SD(2X - Y + 5) =$ $\sqrt{28}$

$Var(2X - Y + 5) = 2^2 Var(X) + (-1)^2 Var(Y) + 2(2)(-1) Cov(X,Y)$

$= 4(3)^2 + 1(4)^2 - 4\, Corr(X,Y)\, SD(X) SD(Y)$

$= 36 + 16 - 4(\tfrac{1}{2})(3)(4)$

$= 52 - 24$

$= 28$

$\Rightarrow SD(X) = \underline{\underline{\sqrt{28} \cong 5.3}}$

2. Suppose $M_X(t)$, the moment generating function of the random variable X, is given by:

$$M_X(t) = \exp\{\,\theta\,[\exp(t) - 1]\,\}.$$

where $\theta > 0$ is an unknown parameter. By direct calculation, evaluate:
Note: Even if you recognize the distribution of X from the form of $M_X(t)$, you are to do the following evaluations using only $M_X(t)$.

[3] a) E(X) = $\frac{d}{dt} M_X(t) \Big|_{t=0}$ ________ θ ________

$$\frac{d}{dt} M_X(t) = \exp\{\theta[\exp(t)-1]\} \cdot \frac{d}{dt}\{\theta[\exp(t)-1]\}$$

$$= \exp\{\quad\} \cdot \theta \exp(t)$$

$$\Rightarrow \frac{d}{dt} M_X(t)\Big|_{t=0} = \exp\{0\} \cdot \theta \cdot 1 = \underline{\underline{\theta}}$$

[4] b) Var(X) = $\frac{d^2}{dt^2} M_X(t)\Big|_{t=0} - [E(X)]^2$ done above ________ θ ________

$$\frac{d}{dt}\left\{\frac{d}{dt} M_X(t)\right\} = \exp\{\quad\} \cdot \frac{d}{dt}[\theta \exp(t)] + \left[\frac{d}{dt} \exp\{\quad\}\right] \cdot \theta \exp(t)$$

$$= \exp\{\quad\}\, \theta \exp(t) + \exp\{\quad\} \cdot [\theta \exp(t)]^2$$

$$\Rightarrow \frac{d^2}{dt^2} M_X(t)\Big|_{t=0} = 1 \cdot \theta + 1 \cdot \theta^2 = \theta + \theta^2$$

$$\Rightarrow \mathrm{Var}(X) = \theta + \theta^2 - \theta^2 = \underline{\underline{\theta}}$$

from (a)

3. Suppose X is a random variable with density function $f_X(x)$ given by:

$$f_X(x) = 2x \qquad \text{for } 0 \le x \le 1.$$

[2] a) $E(X) =$ $\int x f_X(x)\,dx$ $\quad$ $2/3$

$$= \int_{-\infty}^{0} 0\,dx + \int_0^1 x\, 2x\,dx + \int_1^{\infty} 0\,dx$$

$$= 2\left[\frac{x^3}{3}\Big|_0^1\right] = 2/3$$

[3] b) $\mathrm{Var}(X) =$ $E(X^2) - [E(X)]^2$ $\quad$ $1/18$

$$E(X^2) = \int x^2 f_X(x)\,dx$$

$$= \int_0^1 x^2 (2x)\,dx$$

$$= 2\int_0^1 x^3\,dx$$

$$= 2\left[\frac{x^4}{4}\Big|_0^1\right] = \frac{1}{2}$$

$$\Rightarrow \mathrm{Var}(X) = \frac{1}{2} - \left(2/3\right)^2$$

$$= \frac{1}{2} - \frac{4}{9} = \frac{9-8}{18} = \frac{1}{18}$$

[6] c) Now suppose $X_1, X_2, \ldots, X_{90}$ are independent random variables, each having the same distribution as X (as given above) and let $T = X_1 + X_2 + \ldots + X_{90}$.
The approximate value of $P(T > 65)$ = $\underline{0.013}$

$$E(T) = 90 \cdot E(X) = 90(2/3) = 60$$

$Var(T) = 90 \cdot Var(X)$ as X_i's are independent

$$= 90\left(\frac{1}{18}\right)$$

$$= 5$$

But $T \approx N(n \cdot E(X),\ n \cdot Var(X))$ by CLT
provided n is large enough.

$$\Rightarrow P(T > 65) = P\left(\frac{T-60}{\sqrt{5}} > \frac{65-60}{\sqrt{5}}\right)$$

$$\cong P\left(Z > \frac{5}{\sqrt{5}}\right)$$

$$= P(Z > \sqrt{5})$$

$$= P(Z > 2.236)$$

$$\cong 1 - 0.9873 = 0.0127$$

$$\cong \underline{\underline{0.013}}$$

THE END