QUIZ # 2

Statistics 305

Term 2, 2006-2007

Tuesday, February 6, 2007

Time: 2:00pm – 2:30pm

Student Name (**Please print in caps**): _____

Student Number: _____

Notes:

- This quiz has 3 problems on the 4 following pages, plus 1 page of statistical tables. Check to ensure that you have a complete paper.
- The amount each part of each question is worth is shown in [ ] on the left-hand side of the page.
- Where appropriate, record your answers in the blanks provided on the right-hand side of the page.
- Your **solutions must be justified**; show all the work and state all the reasons leading to your answer for each question in the space provided immediately under the question.
- Clear and complete solutions are important; little partial credit will be given.
- This is a closed book exam.
- A **single one-sided 8.5 x 11 page** of notes is allowed.
- Calculators are allowed (but not for symbolic differentiation or integration).
- No devices (including calculators) that can store text or send/receive messages are allowed.

Problem	Total Available	Score
1.	5	
2.	11	
3.	9	
Total	25	

1. Suppose a measurement has a mean of μ and a standard deviation of $\sigma = 3$. We will estimate the value of μ by $\overline{X}$, the average of n such independent measurements.

[5] How large a value of n is required to be 80% confident that this estimate will be within 0.1 of the true value; that is, that $P(|\overline{X} - \mu| \leq 0.1) = 0.80$? _____

2. Suppose X is a positive random variable with a mean of μ_X and a variance of σ_X^2. If $Y = -\ln(X)$, where $\ln \equiv \log_e$, use the delta method to obtain expressions (entirely in terms of μ_X and σ_X^2) for:

Note: You do <u>not</u> need to derive the basic results for the delta method as done in class but you must state clearly the exact form of the results you are using.

[2] a) a "first-order" approximation to $E(Y)$. _____

[4] b) a "first-order" approximation to $SD(Y)$. _____

[3] c) a "second-order" approximation to $E(Y)$. _____

[2] d) If X was uniformly distributed on [0,1], do you expect the delta method approximations to the moments of $Y = -\ln(X)$ to be fairly accurate? Explain. _____

3. Suppose X and Y are bivariate normally distributed random variables with means μ_X, μ_Y, variances σ_X^2, σ_Y^2, and correlation ρ_{XY}. Then $M_{X,Y}(s,t)$, the joint moment generating function of X and Y evaluated at s and t, is given by:

$$M_{X,Y}(s,t) = \exp(s\mu_X + t\mu_Y + [s^2\sigma_X^2 + 2st\rho_{XY}\sigma_X\sigma_Y + t^2\sigma_Y^2]/2).$$

Let $W = aX + bY + c$, where a, b and c are constants.

[6] a) Evaluate $M_W(t)$, the moment generating function of W evaluated at t. _____

[3] b) What is the distribution of W? _____

Note: Explain clearly how this result follows from a).

THE END