QUIZ # 3

Statistics 305

Term 2, 2006-2007

Tuesday, February 27, 2007

Time: 2:00pm – 2:30pm

Student Name (**Please print in caps**): _____

Student Number: _____

Notes:

- This quiz has 3 problems on the 4 following pages, plus 1 page of statistical tables. Check to ensure that you have a complete paper.
- The amount each part of each question is worth is shown in [ ] on the left-hand side of the page.
- Where appropriate, record your answers in the blanks provided on the right-hand side of the page.
- Your solutions **must be justified**; show **all the work** and state **all the reasons** leading to your answer for each question in the space provided immediately under the question.
- Clear and complete solutions are essential; little partial credit will be given.
- This is a closed book exam.
- A single one-sided 8.5 x 11 page of notes is allowed.
- Calculators are allowed (but not for symbolic differentiation or integration).
- No devices (including calculators) that can store text or send/receive messages are allowed.

Problem	Total Available	Score
1.	6	
2.	7	
3.	12	
Total	25	

1. Consider $T = T(X_1, X_2, ..., X_n)$, any estimator of the parameter θ. As discussed in class, the mean squared error and bias of T are defined as $MSE(T) = E[(T-\theta)^2]$ and $Bias(T) = E(T) - \theta$.

[4] a) Show that $MSE(T) = Var(T) + [Bias(T)]^2$. _____

[2] b) Explain the practical importance of both terms in the above expression for $MSE(T)$ when evaluating the performance of the estimator T. _____

2. Suppose Y has a binomial distribution with parameters n and θ; that is,

$$P(Y = y) = \binom{n}{y}\theta^y(1-\theta)^{n-y} \qquad \text{for } y = 0,1,...,n.$$

Assume the value of n is known but θ is an unknown parameter.

[5] a) Find the expression for $\hat{\theta}_{ML}$, the maximum likelihood estimator (MLE) of θ based on the single binomial random variable Y. _____

[2] b) Provide a rough sketch of the likelihood function for $n = 10$ and $y = 5$. _____

3. Suppose $X_1, X_2, \ldots, X_n$ is a simple random sample from an exponential distribution with a rate of θ; that is, from the density function:

$$f_\theta(x) = \theta \exp(-\theta x) \qquad \text{for } x > 0.$$

[3] a) Find the expression for $\hat{\theta}_{MM}$, the method of moments estimator (MME) of θ. _____

[5] b) Find an approximate expression for the variance of the MME $\hat{\theta}_{MM}$. _____

[4] c) Find a second-order approximate expression for the bias of the MME $\hat{\theta}_{MM}$. _____

THE END