QUIZ # 4

Statistics 305

Term 2, 2006-2007

Tuesday, March 13, 2007

Time: 2:00pm – 2:30pm

Student Name (**Please print in caps**): _____

Student Number: _____

Notes:

- This quiz has 3 problems on the 5 following pages, plus 1 page of statistical tables. Check to ensure that you have a complete paper.
- The amount each part of each question is worth is shown in [ ] on the left-hand side of the page.
- Where appropriate, record your answers in the blanks provided on the right-hand side of the page.
- Your solutions **must be justified**; show **all the work** and state **all the reasons** leading to your answer for each question in the space provided immediately under the question.
- Clear and complete solutions are essential; little partial credit will be given.
- This is a closed book exam.
- A single one-sided 8.5 x 11 page of notes is allowed.
- Calculators are allowed (but not for symbolic differentiation or integration).
- No devices (including calculators) that can store text or send/receive messages are allowed.

Problem	Total Available	Score
1.	6	
2.	11	
3.	8	
Total	25	

1. Suppose X and Y are bivariate normally distributed random variables with means $\mu_X = 1$, $\mu_Y = 2$, standard deviations $\sigma_X = 2$, $\sigma_Y = 3$, and correlation $\rho_{XY} = 0.5$.

[6] Evaluate $P(3X - Y > 9) =$ _____

Note: Be sure to indicate clearly all the steps in your evaluation.

2. Consider the exponential distribution with rate θ $(\theta > 0)$; that is, the density function:

$$f_\theta(x) = \theta \exp(-\theta x) \qquad \text{for } x > 0.$$

[3] a) Find the expression for the Fisher Information for θ in a single observation X from this distribution. _____

[3] b) Find the expression for $\hat{\theta}_{ML}$, the maximum likelihood estimator (MLE) of θ based on the simple random sample $X_1, X_2, \ldots, X_n$. _____

[2] c) What is the asymptotic distribution of the MLE $\hat{\theta}_{ML}$? _____

[3] d) What is the form of the approximate 90% confidence interval for the parameter θ based on the MLE $\hat{\theta}_{ML}$? _____

3. Suppose $X_1, X_2, \ldots, X_n$ is a simple random sample from a $N(\theta, 1)$ population. Suppose that the target of inference is not θ but the new parameter $\psi = P(X > 0)$, where X is a single observation from this population.

[2] a) Find an expression for the new parameter ψ in terms of the parameter θ. _____

[2] b) Find $\hat{\psi}_{ML}$, the maximum likelihood estimator (MLE) of ψ based on the simple random sample $X_1, X_2, \ldots, X_n$. _____

Note: You may take as given that $\hat{\theta}_{ML} = \overline{X}$.

[4] c) Find an expression for the asymptotic standard error of $\hat{\psi}_{ML}$. _____

THE END