QUIZ # 5

Statistics 305

Term 2, 2006-2007

Thursday, March 29, 2007

Time: 2:00pm – 3:00pm

Student Name (**Please print in caps**): _____

Student Number: _____

Notes:

- This quiz has 3 problems on the 5 following pages, plus 3 pages of statistical tables. Check to ensure that you have a complete paper.
- The amount each part of each question is worth is shown in [ ] on the left-hand side of the page.
- Where appropriate, record your answers in the blanks provided on the right-hand side of the page.
- Your solutions **must be justified**; show **all the work** and state **all the reasons** leading to your answer for each question in the space provided immediately under the question.
- Clear and complete solutions are essential; little partial credit will be given.
- This is a closed book exam.
- A single one-sided 8.5 x 11 page of notes is allowed.
- Calculators are allowed (but not for symbolic differentiation or integration).
- No devices (including calculators) that can store text or send/receive messages are allowed.

Problem	Total Available	Score
1.	7	
2.	9	
3.	9	
Total	25	

1. Suppose $X_1, X_2, \ldots, X_n$ is a simple random sample from a normal population with a mean of μ and variance of σ^2, where both parameters are unknown.

[4] a) Derive the form of the **exact** $1-\alpha$ confidence interval for the population standard deviation σ.

[3] b) Suppose a simple random sample of $n = 5$ from this population leads to a sample average of $\bar{x} = 22.1$ and a sample standard deviation of $s = 3.7$. Evaluate the **exact** 90% confidence interval for the population standard deviation σ.

2. Suppose $X_1, X_2, \ldots, X_n$ is a simple random sample from a population having an exponential distribution with mean θ ($\theta > 0$); that is, having the density function:

$$f_\theta(x) = \frac{1}{\theta}\exp\left(-\frac{x}{\theta}\right) \quad \text{for } x > 0.$$

[3] a) Find the expression for $\hat{\theta}_{ML}$, the maximum likelihood estimator (MLE) of θ.

[3] b) Show that the MLE $\hat{\theta}_{ML}$ is unbiased and evaluate its **exact** variance.

[3] c) Is there any unbiased estimator of the parameter θ that has smaller variance than the MLE $\hat{\theta}_{ML}$? Be sure to explain your reasoning clearly.

3. Suppose $X_1, X_2, \ldots, X_n$ is a random sample of size n from a $N(\mu, 1)$ population.

[4] a) Use the Neyman-Pearson Lemma to find the form of the most powerful test for testing $H_0 : \mu = 0$ versus $H_1 : \mu = 1$.

[2] b) Write down the explicit form of this test giving the critical value required to achieve a significance level of $\alpha = 0.01$.

[3] c) Suppose we use this same test for testing $H_0 : \mu \le 0$ versus $H_1 : \mu > 0$. Evaluate the power function of this test and show it is monotonically increasing in μ. Sketch the power function as a function of μ (roughly).

THE END