Lab 1: Introductory Exercises in R and Calculus - Solutions

- **Question 1** *Answer*:

```
> x <- seq(1, 400, by = 3)

 [1]   1   4   7  10  13  16  19  22  25  28  31  34  37  40  43  46  49  52
 [19]  55  58  61  64  67  70  73  76  79  82  85  88  91  94  97 100 103 106
 [37] 109 112 115 118 121 124 127 130 133 136 139 142 145 148 151 154 157 160
 [55] 163 166 169 172 175 178 181 184 187 190 193 196 199 202 205 208 211 214
 [73] 217 220 223 226 229 232 235 238 241 244 247 250 253 256 259 262 265 268
 [91] 271 274 277 280 283 286 289 292 295 298 301 304 307 310 313 316 319 322
[109] 325 328 331 334 337 340 343 346 349 352 355 358 361 364 367
370 373 376 [127] 379 382 385 388 391 394 397 400

> length(x)

[1] 134

> x[seq[2,134,by=2]]

[1]   4  10  16  22  28  34  40  46  52  58  64  70  76  82  88  94
100 106 112 [20] 118 124 130 136 142 148 154 160 166 172 178 184 190
196 202 208 214 220 226 [39] 232 238 244 250 256 262 268 274 280 286
292 298 304 310 316 322 328 334 340 [58] 346 352 358 364 370 376 382
388 394 400

> x[x %% 7==0]

[1]   7  28  49  70  91 112 133 154 175 196 217 238 259 280 301 322
343 364 385
```

- **Question 2**

  Answer:

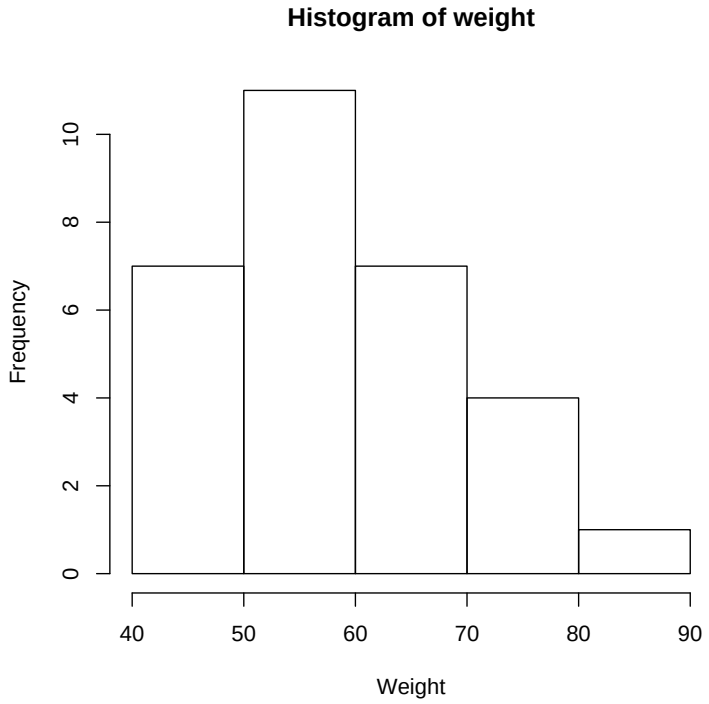
```
> w <- read.csv("lab-1.csv")
> summary(w[, "height"])
```

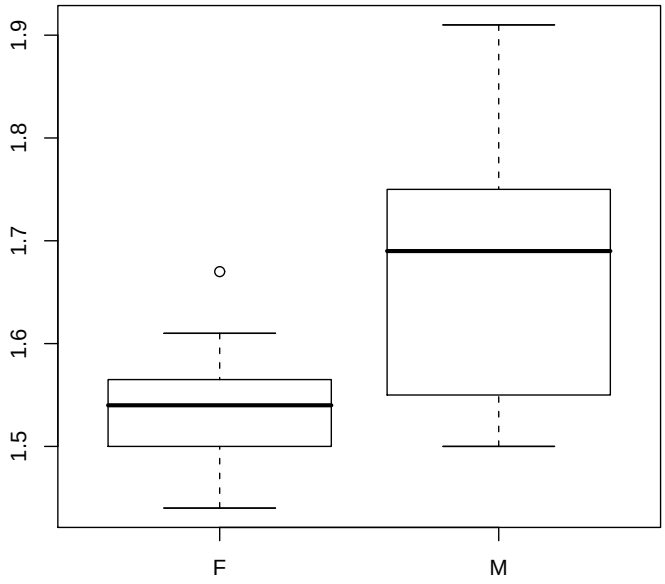
```
 Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 1.440   1.525   1.550   1.603   1.685   1.910
```

To get the variance:

```
>var(w[, "height"])
```

```
> hist(w[, "weight"])
> boxplot(w[, "height"] ~ w[, "Gender"])
```





- **Question 3**
  Answer:
  a) Let $f(x) = sin(x)$. Then $f^{(2k+1)}(x) = (-1)^k cos(x)$, and $f^{(2k)}(x) = (-1)^k sin(x)$ for all k. Then the four term expansion is given by

$$sin(x) = 0 + \frac{x}{1!} + (0) \times \frac{x^2}{2!} + (-1)\frac{x^3}{3!}$$

  and more generally

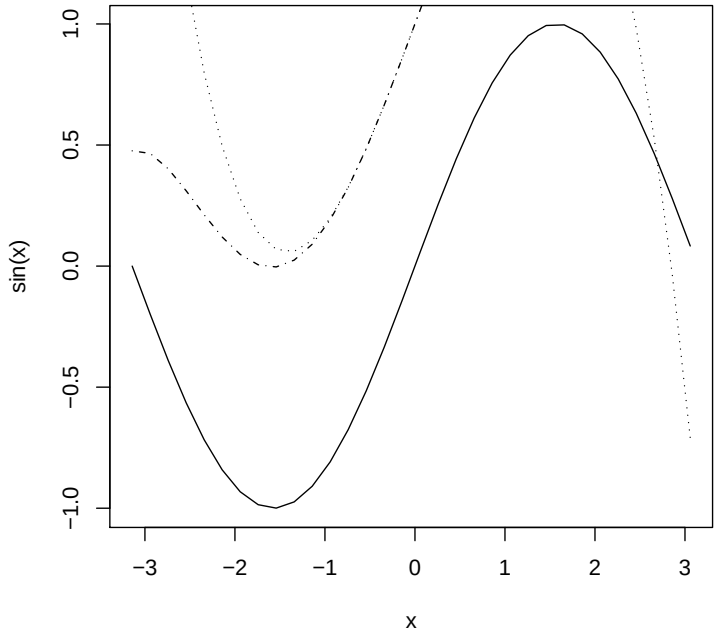
$$sin(x) = \sum_{i=0}^{\infty} (-1)^i \frac{x^{2i+1}}{(2i+1)!}$$

  b)

```
> x <- seq(-1, 1, by = 0.2)
> plot(x, sin(x), type = "l")
> points(x, 1 + x - x^3/6, type = "l", lty = 3)
> points(x, 1 + x - x^3/6 + x^5/(24 * 5), type = "l",lty = 4)
```

c)Multiply part a) by x^3.

- **Question 4**



Answers:

a)

$$Letu = \sqrt{x}, then\ tan(u)' = 1 + tan^2(u) \Rightarrow tan(\sqrt{x})' = \frac{1 + tan^2(\sqrt{x})}{2\sqrt{x}}$$

b)

$$\int sin^2(x) = \int \frac{1 - cos(2x)}{2} = \frac{1}{2}(x - \frac{sin(2x)}{2})$$

c) $y = \sqrt{4 - x^2} \Rightarrow y^2 + x^2 = 4$ Hence, the equation is a quarter of a circle of radius 2. The area of such a region is $4\pi/4 = \pi$

d) Let a, b denote the two sides of such rectangle. Then $2(a + b) = 10 \Rightarrow b = 5 - a$. We want to maximize Area$= ab = a(5 - a)$, when

a is between 0 and 5. To maximize $f(a) = a(5-a)$, let us compute $f'(a) = 5 - 2a$. The only critical point is $5/2$ and $f(0) = f(5) = 0$. Hence $5/2$ is the global maximum and the maximum area is $25/4$.