A Comparison of Several Robust Estimators for a Finite Population Mean

Patrick J. Farrell[a] and Matias Salibian-Barrera[b]

[a]Patrick J. Farrell, School of Mathematics and Statistics, Carleton University, 1125 Colonel By Drive, Ottawa, Ontario, K1S 5B6, pfarrell@math.carleton.ca.

[b]Matias Salibian-Barrera, Department of Statistics, University of British Columbia, 333-6356 Agricultural Road, Vancouver, British Columbia, V6T 1Z2, matias@stat.ubc.ca.

Abstract
In survey sampling, ratio and regression estimators are often used to estimate the mean of a finite population. These estimators make use of information on an auxiliary variable that is assumed to be available over the entire population. Generally speaking, the higher the correlation between the response and this auxiliary variable, the more efficient the ratio and regression estimators will be relative to the simple random sample mean. However, these two estimators are quite sensitive to outliers. In the survey sampling context, Chambers (1986) distinguishes between representative and non-representative outliers. The former type of outlier is defined as an observation with similar counterparts in the non-sampled portion of the population, while the latter is unique. Most research on outlier-robust alternatives to the ratio and regression estimators tends to focus on representative outliers. In this paper, we compare via a simulation study the performance of a number of these alternatives under the presence of representative and non-representative outliers, including those based on M-, GM-, and least absolute value L_1 estimators considered by Bassett and Saleh (1994). We also extend MM-estimators (see Yohai 1987) to the survey sampling context, and evaluate their performance as well.

Keywords: M-Type Regression Estimators, Non-representative Outliers, Representative Outliers, Super-population Model, Winsorization.

1. Introduction

It is often the case in survey sampling that interest centres on the estimation of the unknown mean $\overline{Y}$ of a response variable, *Y*, associated with a finite population. An obvious choice of estimator is the mean, $\overline{y}$, of a simple random sample drawn without replacement from the population. However, such an estimator can be extremely variable depending upon the variability of the response variable across the population. Alternatively, provided that information on a positive auxiliary variable *X* that is highly correlated with *Y* is available for all *N* units in the population $U = \{1, \ldots, j, \ldots, N\}$, estimators with greater efficiency than $\overline{y}$ can be considered that acknowledge the association between the response and auxiliary variables.

Two such estimators are the so-called "ratio" and "regression" estimators. Both are based on drawing a bivariate simple random sample of *n* of the (X_j, Y_j) pairs ($j = 1, \ldots, N$) that define the population. Since complete information is available for the auxiliary variable, the population mean and variance $\overline{X}$ and S_X^2 are known. Both the ratio and regression estimators are based on the premise of comparing $\overline{X}$ to the mean of the auxiliary variable, $\overline{x}$, over the sample. Provided that there is a high degree of correlation between the auxiliary and response variables, this comparison would lead to the consideration of an estimator for $\overline{Y}$ that would be derived by adjusting the sample mean of the response variable $\overline{y}$ upwards or downwards depending upon whether $\overline{X}$ was larger or smaller than $\overline{x}$.

Specifically, the ratio estimator for $\overline{Y}$ is given by $\hat{y}_{ratio} = \overline{y}(\overline{X} / \overline{x})$. This is a good estimator for $\overline{Y}$ when it can be assumed that the survey population can be modeled as the realization of a super-population (see Cochran 1977) where the response variable values in the population are assumed to be realizations of the random variables Y_j, $j = 1, \ldots, N$, according to

$$Y_j = X_j\beta + V(X_j)e_j, \tag{1}$$

where $V(X_j) = X_j$, and the e_j are i.i.d. random variables with mean zero and variance σ^2 that are assumed to follow a distribution *f*. This is due to the fact that for a bivariate sample (x_i, y_i) of size *n*, the weighted least squares estimator for β is $\hat{\beta} = \overline{y} / \overline{x}$.

Alternatively, the regression estimator is given by $\hat{y}_{reg} = \overline{y} + b(\overline{X} - \overline{x})$, where *b* is an estimator of the true slope between the response and auxiliary variables. Note that if *b* is assumed constant, the value for *b* that minimizes the variance $V(\hat{y}_{reg})$ is $b^* = \rho\, S_Y / S_X$, where S_Y is the population standard deviation of the response variable, and ρ is the correlation coefficient between the response and auxiliary variables over the entire population. Therefore, the regression estimator will be a good choice for estimating $\overline{Y}$ when the response variable values in the population are assumed to be realizations of Y_j according to

$$Y_j = \alpha + X_j\beta + e_j, \qquad (2)$$

since for a bivariate sample (x_i, y_i) of size n, the least squares estimator for β is $\hat{\beta} = \hat{\rho}\, s_y / s_x$, where $\hat{\rho}$ is an estimate of ρ, and s_y and s_x are sample standard deviations. In other words, when model (2) above holds, $\hat{y}_{reg} = \overline{y} + \hat{\beta}(\overline{X} - \overline{x})$ is optimal among those estimators of the form $\overline{y} + b(\overline{X} - \overline{x})$.

Both the ratio and regression estimators are clearly sensitive to outliers. In the survey sampling context, Chambers (1986) distinguishes between two types of outliers, which he refers to as representative and non-representative. Chambers (1986) defines a representative outlier as "… a sample element with a value that has been correctly recorded and that cannot be regarded as unique". Thus, in this case the non-sampled portion of the population may contain units with similar values; however, these units possess values that differ significantly from those of the majority of the units in the population. By contrast, Chambers (1986) refers to non-representative outlier as one that "… is typically associated with a sample datum that is either incorrect (due, for example, to errors in coding) or is unique to the particular population element involved".

Most research on outlier-robust alternatives to the ratio and regression estimators are proposed tends to focus on representative outliers. In principle, a non-representative outlier can be dealt with using survey editing if it is deemed the result of an error, or, if it is unique, by giving it a weight of one in order to exclude it (see Chambers 1986 and Gwet and Rivest 1992). Consequently, the case of non-representative outliers has not received much attention in the literature. However, if an atypical observation is detected in a sample, it will generally be difficult for the practitioner, if not impossible, to categorize it as representative or non-representative, since knowledge of the response variable distribution over the entire population will not be available. It is therefore important to assess how the classical and the different outlier-robust estimators perform when there are both representative and non-representative outliers in the sample.

Early outlier-robust alternatives that have been developed for the standard-type estimators of a finite population mean are based on winsorization (due to Charles P. Winsor; see Tukey 1962), which consists of replacing large observations by either a pre-determined value, or one that is determined according to the sample data. It is a commonly used technique to reduce the importance of outliers for the location model $Y_j = \mu + e_j$, where μ is a constant. This methodology has been investigated in the survey sampling context by Searls (1966), Ernst (1980), Fuller (1991), and Rivest (1993). Despite the fact that the winsorized sample mean is a biased estimator of the finite population mean, Searls (1966) demonstrated that for skewed populations, it possesses a smaller mean square error than the simple random sample mean.

Noting that both the ratio and regression estimators can be derived from a regression super-population model, an intuitively clear strategy to obtain outlier-robust alternatives to these estimators is to use robust regression estimators. Huber (1973) introduced M-estimators for linear regression models. However, these estimators are still susceptible to the effect of high-leverage outliers. Generalized M-estimators (GM) attempt to control the effect of high-leverage outliers on the regression estimators (see Hill 1977, Krasker 1980, Krasker and Welsch 1982, and Hampel *et al*. 1986). In the survey sampling context, M- and GM-estimators were studied by Bassett and Saleh (1994), Chambers (1986), Gwet and Rivest (1992), and Hulliger (1995), among others.

The breakdown point of an estimator is the largest proportion of arbitrary observations that can be present in a data set before the estimator is driven beyond all bounds (see, for example, Donoho and Huber 1983). Maronna *et al*. (1979) showed that GM-regression estimators have low breakdown point when high-leverage outliers may be present in the sample.

In this paper, we propose to extend MM-regression estimators (see Yohai 1987) to the survey sampling context. In the infinite-population model, these estimators are able to simultaneously achieve high-breakdown point and high efficiency when no outliers are present. Our simulation results indicate that this property extends to the finite population case.

We report the results of a simulation study that compared the performance of outlier-robust alternatives to the ratio and regression estimators based on M-, GM-, and MM-estimators under the presence of representative and non-representative outliers. In Section 2, we describe the outlier-robust estimators that have been proposed in the literature, along with those based on MM-estimators. Section 3 discusses the details surrounding the simulation study, while a conclusion and discussion is given in Section 4.

2. Outlier Robust Alternative Estimators

Note that both the ratio and regression estimators are calculated using a least squares regression estimator. In the case of the former estimator, $\hat{y}_{ratio} = \hat{\beta}\overline{X}$ where $\hat{\beta} = \overline{y} / \overline{x}$, while for the latter $\hat{y}_{reg} = \overline{y} + \hat{\beta}(\overline{X} - \overline{x})$ where $\hat{\beta} = \hat{\rho}\, s_y / s_x$.

Hence, a first approach to obtain robust alternatives for these estimators is to replace the least squares regression estimator $\hat{\beta}$ above by a robust regression estimator.

To fix ideas, assume that the response variable values in the population are realizations of the random variables Y_j, $j = 1, \ldots, N$, according to

$$Y_j = X_j\beta + V(X_j)e_j, \tag{3}$$

where the e_j are as defined in (1) above, and $V(X_j)$ is a known function. It is easy to see that in this case the least squares estimator b_n satisfies

$$b_n = \min_b \sum_{i=1}^{n} \left(\frac{y_i - bx_i}{\sigma\sqrt{V(x_i)}} \right)^2 .$$

Let ρ be an even and non-decreasing loss function in $(0, \infty)$ such that $\rho(0) = 0$. The associated regression M-estimator b_n (see Huber 1973) solves

$$b_n = \min_b \sum_{i=1}^{n} \rho\left(\frac{y_i - bx_i}{\hat{\sigma}\sqrt{V(x_i)}} \right),$$

or equivalently

$$\sum_{i=1}^{n} \psi\left(\frac{y_i - b_n x_i}{\hat{\sigma}\sqrt{V(x_i)}} \right)\left(\frac{x_i}{\sqrt{V(x_i)}} \right) = 0,$$

where $\psi(u) = \rho'(u)$ and $\hat{\sigma}$ is a robust estimator of the error scale in $V(y_i) = \sigma^2 V(x_i)$.

Unfortunately, M-estimators are not robust against high-leverage outliers (Maronna *et al*. 1979). To address this problem, GM-estimators (see Hill 1977, Krasker 1980, Krasker and Welsch 1982, and Hampel *et al*. 1986) down-weight the role of high-leverage observations in the estimating equations by incorporating a weight function $w(x)$. Specifically, they are defined as the solution to

$$\sum_{i=1}^{n} \psi\left(\frac{y_i - b_n x_i}{\hat{\sigma}\sqrt{x_i}\, w(x_i)} \right)\left(\frac{x_i w(x_i)}{\sqrt{x_i}} \right) = 0,$$

for a particular choice of weight function $w(x)$. One such choice would be the distance between x and a location parameter μ_X for the auxiliary variable X.

Gwet and Rivest (1992) suggested an outlier-robust alternative to the ratio estimator that is based on an M- or a GM-estimator. They showed that if the score function $\psi(u)$ is monotone increasing (which corresponds to an unbounded convex loss function ρ) and the covariates are strictly positive, then the resulting estimators are asymptotically design-consistent (see Wright 1983). Using a first-order linearization of the estimating equations, they were able to calculate the asymptotic bias of the population mean estimator, and also the finite-population equivalent of the influence function of the regression estimator (see Hampel *et al*. 1986). They also conducted a Monte Carlo study using two different populations containing outliers in order to compare their estimators to, among others, those of Chambers (1986), and the standard ratio estimator. Their results demonstrate that the mean square error of the outlier-robust estimators can be substantially smaller than that of the ratio estimator.

Bassett and Saleh (1994) proposed an outlier-robust methodology for estimating the population median of a response variable under the assumption of complete knowledge of an auxiliary variable. Their estimator was based on the super-population model defined in (3). However, rather than using weighted least squares to estimate β, Bassett and Saleh (1994) suggest using the least absolute value (L_1) estimate based on a bivariate simple random sample instead. The estimator for the population median of the response variable is then specified simply as the product of the L_1 estimator for β and the population median of the auxiliary variable, which is known.

Hulliger (1995) developed design-based outlier-robust M-estimators for a finite population mean that were based on data obtained via unequal probability sampling. Specifically, the simple linear model that implicitly underlies the Horvitz-Thompson (HT) estimator is explicitly expressed as a least squares functional of an empirical distribution function that acknowledges the complexity of the sample design. This leads to a straightforward robustification of the

HT estimator. Hulliger (1995) also developed an adaptive version of this robustified estimator, and demonstrated via a simulation study that his proposed estimators outperformed the standard HT estimator in many outlier situations.

Unfortunately, M-type estimators with unbounded loss function ρ have low breakdown point against high-leverage outliers (see Maronna *et al*. 1979). High-breakdown point robust regression estimators include S-estimators (see Rousseeuw and Yohai 1984). These estimators are defined as the vector of regression coefficients that minimizes a robust M-scale estimator of the residuals. Specifically, for each b, define the scale $\sigma(b)$ as the solution to

$$\frac{1}{n}\sum_{i=1}^{n}\rho\left(\frac{y_i - bx_i}{\sigma(b)\sqrt{V(x_i)}}\right) = a,$$

where $0 < a < 1$ and $\rho(u)$ is an even, non-decreasing function in $(0, \infty)$ such that $\rho(0) = 0$. The S-regression estimator b_n is defined as

$$b_n = arg\ min_b\ \sigma(b)$$

Associated with the S-regression estimator is a robust scale estimator of the residuals, namely $\hat{\sigma}_n = \sigma(b_n)$. The constant a determines the consistency and the breakdown point of the S-estimator, which is $\min(a, 1 - a)$. It follows that $a = 1 / 2$ gives the highest possible breakdown point. The function ρ is then chosen such that $E\{\rho(u/\sigma)\} = 1/2$ where $u \sim N(0, \sigma^2)$. Unfortunately, high-breakdown S-estimators have low efficiency when the data do not contain outliers (Rousseeuw and Leroy 1987). To obtain simultaneous high-breakdown point and high-efficiency, Yohai (1987) proposed the class of MM-estimators that are defined as the local minimum $\tilde{b}_n$ of

$$h(b) = \sum_{i=1}^{n}\rho_1\left(\frac{y_i - bx_i}{\hat{\sigma}_n\sqrt{x_i}}\right),$$

such that $h(\tilde{b}_n) \le h(\hat{b}_n)$, where $\hat{b}_n$ is the S-regression estimator associated with the S-scale $\hat{\sigma}_n$. Moreover, the function ρ_1 satisfies the same regularity conditions as $\rho(u)$ and additionally $\rho_1(u) \le \rho(u)$ and $\sup_u \rho_1(u) = \sup_u \rho(u)$, where ρ is the function used for the S-estimator. In what follows, we use functions $\rho(u)$ in Tukey's bi-square family (see Beaton and Tukey 1974); in particular, we use $\rho_d(u) = 3(u/d)^2 - 3(u/d)^4 + (u/d)^6$ for $|u| \le d$ and $\rho_d(u) = 1$ for $|u| > d$, where d is a tuning constant, which can be chosen (for ρ and ρ_1) to obtain the desired breakdown point and efficiency.

We conjecture that the approach used in Gwet and Rivest (1992) to prove the asymptotic design-consistency of the estimators based on a GM-estimator can be extended to S- and MM-estimators. Although our numerical simulation results are encouraging, this question is beyond the scope of this paper.

Rather than replacing the least squares regression estimator by a robust alternative in the formulae for the ratio and regression estimators given above, Chambers (1986) proposed an estimator that maintains the prediction error relatively stable under the presence of outliers. Specifically, for a robust estimator b_n and a function $\psi(u)$, he proposed an estimator $\hat{t}_n$ for the population total of the form

$$\hat{t}_n = \sum_{i\in s} y_i + b_n \sum_{j\notin s} X_j + \sum_{i\in s} u_i \psi[(y_i - b_n x_i)/\sigma_i]. \tag{4}$$

In this expression, $\sigma_i = V(x_i)\sigma^2$, the sum $i \in s$ is over all sampled units, while the sum $j \notin s$ is over all non-sampled population units. In addition, the quantity $u_i = x_i\sigma_i^{-1}\sum_{j\notin s} X_j / \sum_{i\in s} x_i^2\sigma_i^{-2}$. The robustness of the estimator in (4) will depend upon the choice of b_n and ψ. Typically, the former will be a robust and efficient estimator of β. The choice of the real-valued function ψ is more difficult. However, it should be bounded and skew-symmetric so that $\psi(-t) = -\psi(t)$. In addition, from the point of view of efficiency under the super-population model (3), it must also satisfy $\lim_{|t|\to 0} \psi(t)/t = 1$. Under certain regularity conditions, Chambers (1986) finds the asymptotic bias and variance of the associated population total estimators under a gross-error contamination model. Furthermore, to assess the performance of the estimator in (4) for different choices of ψ in practice, Chambers (1986) conducted an intensive simulation study involving numerous estimators, and a variety of sample designs. A study population consisting of 557 census blocks in east metropolitan Baltimore was used where the response variable was defined to be the 1970 census count of the total population in each block, and the auxiliary variable was the corresponding 1960 census count of the number of occupied dwellings in each block. Even with the assumption of super-population model (3) with $V(X_j) = X_j$, it was deemed that outliers were present in the population. The results of the simulation conducted by Chambers (1986) suggested that an estimator $\hat{t}_n$ based on a function

$$\psi(t) = t[\exp\{-(a/2)(|t| - b)^2\}],$$

in which a = 0.5 and b = 6 had the smallest root mean square error for all sampling schemes considered.

3. Simulation Study

In this section, we discuss a simulation study that was conducted in order to compare the performance of a number of the standard and robustified estimators for a finite population mean when representative or non-representative outliers exist. These populations were created using three versions of the generic super-population model

$$Y_j = X_j\beta + V(X_j)e_j, \tag{5}$$

where β was set as 1.7, the X_j were assumed to follow a beta distribution with both parameters equal to 0.1, while the e_j were assumed to be normally distributed with a mean of zero. The features that distinguished the three versions of the super-population model were restricted to the function specified for $V(X_j)$, and the variance proposed for the e_j. For Version I, $V(X_j) = 1$ while the standard deviation of the e_j, denoted by σ, was set at 0.75. In Version II, $V(X_j) = X_j$ and σ = 0.75, while for Version III, $V(X_j) = X_j^2$ and the standard deviation σ = 0.10.

These three versions of the super-population model given in (5) were used to create finite populations of size N = 600. In what follows, we refer to these three finite populations created using Versions I through III of the model given in (5) as I(a), II(a), and III(a) respectively. Note that all three reflect a relationship between the two variables that passes through the origin. However, for the population based on Version I of (5), the variance of the response variable, $V(Y_j)$, is constant. By contrast, for the populations generated using Versions II and III of (5), $V(Y_j)$ increases as the value of the auxiliary variable increases; for Version II the increase is linear, while in Version III, the increase is proportional to the square of X_j. The distribution of the response variable for each of the three finite populations is illustrated graphically on the left hand side of Figure 1. For the population created using Version I of the model in (5), the distribution of Y_j is unimodal, symmetric, and bell-shaped. The finite population generated under Version II possesses a distribution for the response variable that is right skewed, while the distribution of Y_j for the population created using Version III is right-skewed and bimodal.

Populations I(a), II(a), and III(a) do not contain outliers. Random samples from these populations were contaminated by the addition of 10% or 20% of low- or high-leverage outliers. These samples thus contain outliers that are non-representative. Note that we also intend to select samples from contaminated populations to investigate the behaviour of the estimators considered in this study when the samples contain representative outliers (as the outliers present in the samples correspond to actual observations in the finite populations being sampled). To accomplish this, four additional populations were created from each of Populations I(a) and II(a) by randomly replacing 5% and 10% of the observations by low- and high-leverage outliers. For example, a finite population I(b) with N = 600 was obtained by randomly substituting 5% of the units in population I(a) with low-leverage outliers around (X_j, Y_j) = (0.5, 5). The outliers were randomly generated by following a bivariate normal distribution with independent co-ordinates with mean (0.5, 5) and standard deviations equal to 0.1. Similarly, by replacing 5% of the units in population I(a) with high-leverage outliers around (3, 20), a population I(c) was created. Populations I(d) and I(e) were obtained analogously to I(b) and I(c) respectively; however these two populations consisted of 10% low- and high-leverage outliers as described above. Counterpart populations II(b) through II(e) were obtained in an identical manner using II(a). Finally, a finite population IV(a) was constructed to mimic the population in Figure 2 of Gwet and Rivest (1992). It consists of N = 235 observations with thirteen outliers. For comparative purposes with populations I(a), II(a), and III(a) that do not contain outliers, the right hand side of Figure 1 presents the distribution of the response variable for I(e), II(e), and IV(a).

A total of 5,000 samples of size n = 60 were drawn from each of the populations I(a) through I(e), II(a) through II(e), and III(a). This process was then repeated for samples of size n = 30. To be consistent with Gwet and Rivest (1992), 5,000 samples of sizes n = 10, 20, and 40 were selected. For each sample, estimates for the finite population mean based on eight different estimators were calculated. Specifically, these estimators were

LS-1: $\hat{y}_{ratio} = \bar{y}(\bar{X} / \bar{x})$

LS-2: $\hat{y}_{reg} = \bar{y} + b(\bar{X} - \bar{x})$

LS-3: $\hat{y}_{reg\_ls\_rob} = \bar{y}_{rob} + b(\bar{X} - \bar{x}_{rob})$

MM-1: $\hat{y}_{ratio\_rob} = \bar{y}(\bar{X} / \bar{x}_{rob})$

MM-2: $\hat{y}_{reg\_rob} = \bar{y}_{rob} + b_{rob}(\bar{X} - \bar{x}_{rob})$

L1-1: $\hat{y}_{reg\_L1} = \bar{y} + b_{L1}(\bar{X} - \bar{x})$

L1-2: $\hat{y}_{reg\_L1\_rob} = \bar{y}_{rob} + b_{L1}(\bar{X} - \bar{x}_{rob})$

CH: $\hat{y}_{Chamb} = \hat{t}_n / N$ where $\hat{t}_n = \sum_{i \in s} y_i + b_n \sum_{j \notin s} X_j + \sum_{i \in s} u_i \psi[(y_i - b_n x_i)/\sigma_i]$

Note that LS-1 and LS-2 are simply the standard ratio and regression estimators, where $\bar{X}$ is the population mean of the auxiliary variable, $\bar{x}$ and $\bar{y}$ are sample means of the auxiliary and response variables, and b is the least squares estimator of β that is based on a model containing an intercept term. The estimates $\bar{x}_{rob}$ and $\bar{y}_{rob}$ in LS-3 are robust M-estimators of the centre of the samples. Specifically, for a tuning constant $c > 0$, let

$$\Psi_c(u) = min\,[max\,(-c, u), c],$$

be Huber's score function (see Huber 1964) and let $\bar{x}_{rob}$ be the solution to

$$\frac{1}{n}\sum_{i=1}^{n} \psi_c\left(\frac{x_i - \bar{x}_{rob}}{\hat{\sigma}}\right) = 0\ ,$$

where $\hat{\sigma}$ is a robust scale estimator. In our simulation, we used $\hat{\sigma} = median_i \left| x_i - median_j(x_j) \right|$, the median absolute deviation from the median (Rousseeuw and Leroy 1987) and $c = 1.345$. The latter choice was motivated by the fact that, in the infinite population setup, the resulting estimator is 95% efficient when the observations $x_1, \ldots, x_n$ are normally distributed. The location estimator $\bar{y}_{rob}$ was computed analogously with the values of y_i, $i = 1, \ldots, n$.

The estimators MM-1 and MM-2 are robustified M-estimators of the ratio and regression estimators LS-1 and LS-2. In addition to $\bar{x}_{rob}$ and $\bar{y}_{rob}$ used in MM-1, the estimator MM-2 also employs a robust MM-estimator, b_{rob}, of the slope parameter. The tuning constants for $\rho(u)$ in Tukey's bi-square family for the S- and MM- estimators were d = 1.548 and d = 4.685 respectively. In the infinite population setup, these choices yield a regression estimator with both 50% breakdown point and 95% efficiency when the errors are normally distributed. The two estimators L1-1 and L1-2 were based on the L_1 estimator for the slope parameter for β considered by Bassett and Saleh (1994). The estimator L1-2 is simply a robustified version of L1-1, with the M-estimators $\bar{x}_{rob}$ and $\bar{y}_{rob}$ used instead of $\bar{x}$ and $\bar{y}$. Finally, CH is based on Chambers' (1986) estimator given in (4) where b_n is the same MM-estimator used in MM-1, and $\psi(t) = t[\exp\{-(a/2)(|t| - b)^2\}]$ with $a = 0.5$ and $b = 6$ (as recommended in Chambers 1986).

The estimates obtained for each of the 5,000 samples (contaminated or not) drawn from a particular finite population were then used to compute estimates of the relative bias (RB) and relative root mean square error (RRMSE) for each of the eight estimators according to

$$\text{RB} = \left[\sum_{j=1}^{5000} (\hat{y}_j - \bar{Y})/5000\right]/\bar{Y}$$

and

$$\text{RRMSE} = \left[\sqrt{\sum_{j=1}^{5000} (\hat{y}_j - \bar{Y})^2/5000}\ \right]/\bar{Y}$$

respectively, where $\bar{Y}$ is the population mean, and $\hat{y}_j$ is, for a particular estimator, the estimate for the population mean obtained from the j-th sample. Note that there are twelve finite populations in total, labeled I(a) through I(e), II(a) through II(e), III(a), and IV(a).

Table 1 presents the estimates obtained for RB when the eight estimators were used to estimate the mean of population I(a) under the five scenarios of no outlier contamination, and the four different degrees of non-representative outlier contamination described above. Results obtained for both n = 30 and n = 60 are presented. Tables 2 and 3 contain analogous RB estimates for populations II(a) and III(a). For the case of no outlier contamination, the standard least squares estimators LS-1 and LS-2, along with the L_1 estimator L1-1 have the smallest estimates of RB for all three populations. Given some level of non-representative outlier contamination, MM-1 possesses relatively small estimates of RB for all three populations, and generally speaking, seems to perform the best with respect to relative bias. Similar results are obtained for MM-2 when samples from populations II(a) and III(a) are contaminated; however RB estimates are noticeably higher than for MM-1 for population I(a). With the exception of 20% high-leverage outlier contamination, L1-2 performs similarly to MM-1 when non-representative outliers are added to any one of the three populations. The LS-3 and CH estimators perform well for low-leverage outlier contamination. Across the three populations, estimates of RB for the former estimator under this type of contamination are similar to those of MM-1. Analogous low-contamination RB estimates for CH are similar to those of MM-1 under population I(a), notably smaller for population II(a), but larger for population III(a).

Tables 4 through 6 give the counterpart RRMSE estimates of the estimates of RB presented in Tables 1 through 3, respectively. For the case of no outlier contamination, the standard least squares estimators LS-1 and LS-2, along with the L_1 estimator L1-1 have the smallest estimates of RRMSE for populations I(a) and II(a). There is little difference among all estimators for the case of population III(a) when there is no contamination with non-representative outliers. When non-representative outliers are incorporated into population I(a), it would appear that CH possesses the best results with regards to RRMSE, with the exception of 20% high-leverage contamination (and 10% high-leverage with n = 60) , where MM-1 is better. Estimates of RRMSE under this latter estimator are generally similar to those of CH. Also of note for population I(a) is the admirable performance of LS-3 and L1-2 for low-leverage outliers. Similar results regarding the relative performance of the estimators are obtained when population II(a) is contaminated with non-representative outliers. However, one notable difference with population I(a) is the similarity of the RRMSE estimates for MM-1 and MM-2. This similarity holds when outliers are introduced into population III(a), where these two estimators tend to produce the best results with respect to RRMSE, followed by L1-2, and then CH. In addition, relative to the other estimators, LS-3 once again performs well for low-leverage observations. Thus, to summarize, in the presence of non-representative outliers, MM-1 seems to perform relatively well with respect to RRMSE. The performance of CH is also notable; however this estimator did not manage well in the situations where population III(a) was contaminated.

In order to assess the performance of the estimators when representative outliers are present, Table 7 presents the estimates of RB obtained for the eight estimators when estimating the finite population means of populations I(a) through I(e), II(a) through II(e), III(a), and IV(a). With the exception of population IV(a), results are reported for samples of size n = 30 and n = 60. For IV(a), in order to be consistent with Gwet and Rivest (1992), sample sizes of n = 10, n = 20, and n = 40 were used. Regardless of the nature (low versus high leverage, 5% versus 10%) of the representative outliers incorporated into population I(a), the estimates of RB are similar and smallest for LS-3, MM-1, L1-2, and CH. For population II(a), a similar conclusion can be drawn with regards to these four estimators; however, with the exception of 10% high-leverage representative outliers, the estimates for CH seem to be notably smaller than those for LS-3, MM-1, and L1-2. In addition, estimates of RB for MM-2 for populations II(b) through II(e) are close to those associated with these latter three estimators. Finally, for population IV(a), estimates of RB are best for MM-1 and L1-2. However, with the exception of this population, in the presence of representative outliers, CH seems to be the estimator of choice with regards to RB.

The analogous RRMSE estimates obtained for the eight estimators when estimating the finite population means of populations I(a) through I(e), II(a) through II(e), III(a), and IV(a) are given in Table 8. Generally speaking, in the presence of representative outliers, the RRMSE estimates appear to be smallest for CH. Estimates for LS-3, MM-1, L1-2 are better than those obtained for LS-1, LS-2, and L1-1 when populations I(b) through I(e) and II(b) through II(e) are considered; however, they are clearly worse than those of CH. Also worthy of note is the similarity of the MM-1 and MM-2 estimates of RRMSE for populations II(b) through II(e), and the closeness of the estimates for all estimators when population IV(a) is considered. Nevertheless, it would appear that, as was the case with RB, the CH estimator seems to be the one of choice with regards to RRMSE when representative outliers are present.

4. Conclusion and Discussion

A simulation study was conducted in order to evaluate and compare the performance of the standard ratio and regression estimators (denoted by LS-1 and LS-2, respectively) with outlier-robust alternatives in the presence of representative and non-representative outliers. The latter type of unusual observation has received little attention in the literature.

Among the outlier-robust alternatives investigated were two least absolute value L_1 estimators (L1-1 and L1-2) in the spirit of Bassett and Saleh (1994), an estimator proposed by Chambers (1986) referred to as CH, and an outlier-resistant least squares alternative, LS-3. In addition, MM-estimators were extended in this study to the survey sampling context, and two such estimators, MM-1 and MM-2, were also evaluated in the simulation study.

When representative outliers were present in the samples, CH yielded in general the best results with regards to relative bias and relative root mean square error. With regards to relative bias, results obtained for MM-1, L1-2, and LS-3 were also promising, and in many cases very similar to the results associated with CH. In fact, these three estimators were often able to out-perform CH in the presence of high-leverage representative outliers. In addition, CH encountered some difficulty when applied to population IV(a), which is analogous to the one considered by Gwet and Rivest (1992), and was dramatically bettered by MM-1, L1-2, and LS-3. When relative root mean square errors are considered, CH is clearly the estimator of choice from those investigated.

The simulation study also investigated the performance of the estimators in the presence of non-representative outliers. Generally speaking, when relative bias is considered, MM-1, the extended MM-estimator proposed in this study, performs well when compared to the others estimators included in the study. The other MM-estimator, MM-2 also performed well when $V(X_j) \neq 1$. In addition, for low-leverage outlier contamination, L1-2, LS-3, and CH performed similarly to MM-1. Of note is that those of CH were notably smaller than those of MM-1 when $V(X_j)$ is proportional to X_j, but larger when $V(X_j) = X_j^2$. In terms of relative root mean square error, generally speaking MM-1 yields the most promising results. However there are some cases for population I(a) with low-leverage outlier contamination where CH performs slightly better than MM-1.

To summarize, CH and MM-1 seemed to perform relatively well under the conditions dictated by the simulation study. The former estimator appears to be strongest in the presence of representative outliers, while the latter was best for the cases in which non-representative outliers persisted. Clearly, these conclusions are limited to the simulation study considered, and further work is necessary in order to better comprehend the performance of these estimators in the presence of these two different types of outliers. In particular, some theoretical development surrounding the extensions of the MM-estimator, MM-1 and MM-2, seems warranted. This research is however, beyond the scope of the present study.

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Figure 1: Histograms of the response variable distribution in populations I(a), II(a), and III(a) that do not contain outliers, along with populations I(e), II(e), and IV(a).

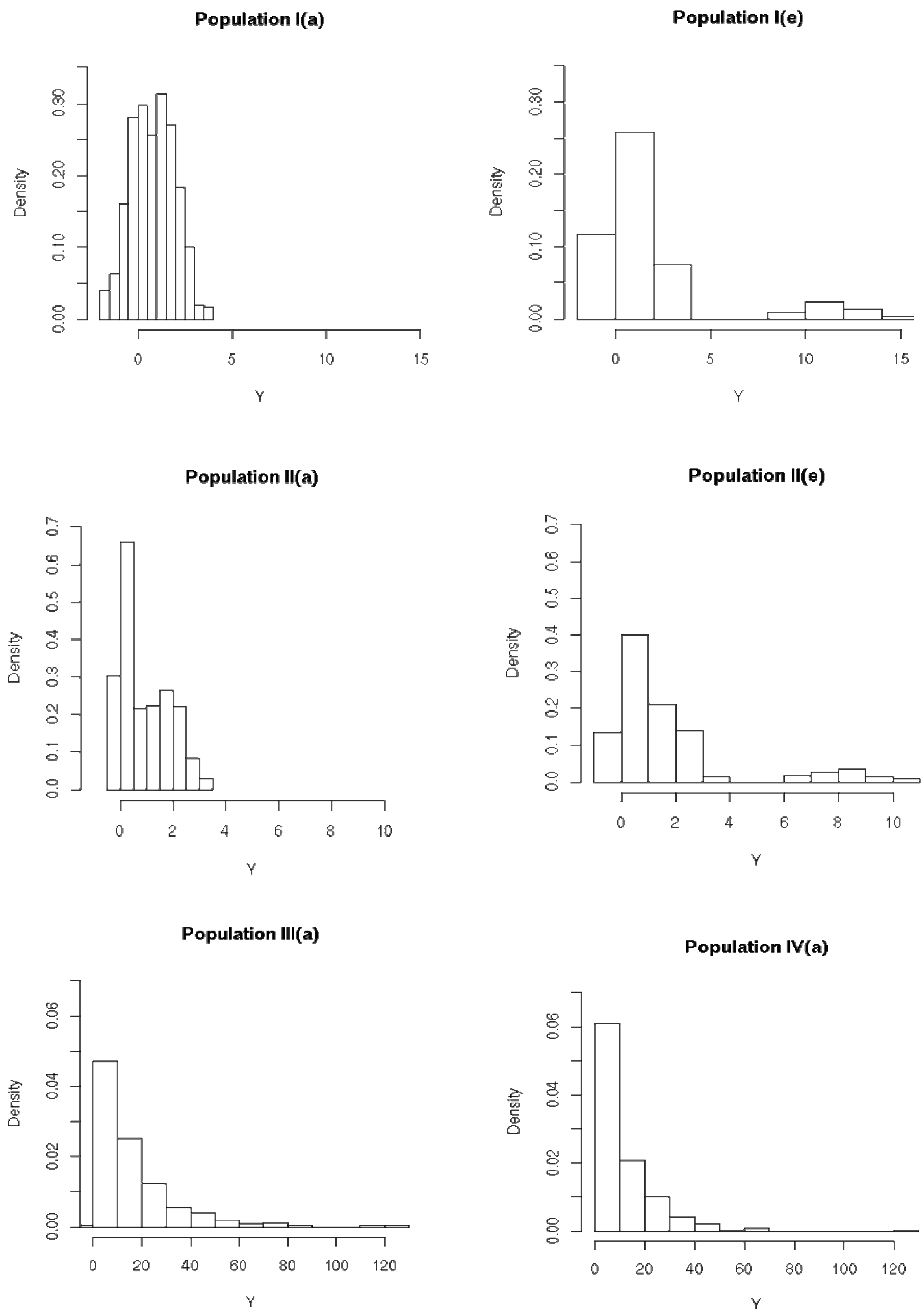

Table 1: For each estimator, estimated relative biases based on 5,000 samples from population I(a), where the samples are contaminated with non-representative outliers.

Outliers	*n*	LS-1	LS-2	LS-3	MM-1	MM-2	L1-1	L1-2	CH
None	30	0.0057	0.0032	0.0380	0.0382	1.0823	0.0059	0.0388	0.0471
None	60	0.0010	0.0000	0.0394	0.0394	0.2892	0.0011	0.0398	0.0427
Low10	30	0.4890	0.4908	0.2972	0.2803	0.6699	0.4844	0.2844	0.2873
Low10	60	0.4808	0.4813	0.2808	0.2736	0.3151	0.4790	0.2754	0.2930
High10	30	0.7711	1.1893	-0.1780	0.1013	0.4954	1.4973	0.0991	0.1401
High10	60	0.7592	1.1800	-0.2496	0.0844	0.1251	1.4957	0.0253	0.2291
Low20	30	0.8895	0.8922	0.5901	0.5759	0.6667	0.8829	0.5780	0.4764
Low20	60	0.8789	0.8793	0.5750	0.5713	0.5747	0.8770	0.5721	0.4830
High20	30	1.0468	1.7126	-1.1350	0.0533	0.1617	0.8002	-1.2766	0.2381
High20	60	1.0382	1.7055	-1.3841	-0.0187	0.0197	0.6992	-1.6136	0.4157

Table 2: For each estimator, estimated relative biases based on 5,000 samples from population II(a), where the samples are contaminated with non-representative outliers.

Outliers	*n*	LS-1	LS-2	LS-3	MM-1	MM-2	L1-1	L1-2	CH
None	30	0.0022	0.0021	-0.1923	-0.2139	-0.1890	0.0011	-0.1950	-0.0237
None	60	0.0005	0.0005	-0.1684	-0.1734	-0.1711	0.0001	-0.1694	-0.0023
Low10	30	0.4330	0.4348	0.0885	0.0694	0.0644	0.4253	0.0768	0.0173
Low10	60	0.4269	0.4278	0.1263	0.1212	0.1127	0.4235	0.1222	0.0567
High10	30	0.8695	1.0546	-0.3187	-0.1047	-0.0891	1.4363	-0.1202	0.0787
High10	60	0.8606	1.0473	-0.3031	-0.0606	-0.0540	1.4333	-0.0845	0.1786
Low20	30	0.7905	0.7931	0.4094	0.3926	0.3812	0.7790	0.3961	0.0565
Low20	60	0.7818	0.7831	0.4404	0.4341	0.4281	0.7763	0.4349	0.1062
High20	30	1.2001	1.5277	-0.9665	-0.0894	-0.0605	2.4722	-0.2386	0.1745
High20	60	1.1934	1.5218	-1.0984	-0.1064	-0.0639	2.4723	-0.2820	0.3470

Table 3: For each estimator, estimated relative biases based on 5,000 samples from population III(a), where the samples are contaminated with non-representative outliers.

Outliers	*n*	LS-1	LS-2	LS-3	MM-1	MM-2	L1-1	L1-2	CH
None	30	-0.0006	-0.0011	-0.0582	-0.0636	-0.0666	0.0011	-0.0604	-0.0219
None	60	-0.0004	-0.0008	-0.0568	-0.0643	-0.0660	0.0002	-0.0600	-0.0273
Low10	30	0.3867	0.3769	0.0408	-0.0197	-0.0235	0.3743	-0.0122	0.0319
Low10	60	0.5317	0.5255	0.0560	0.0079	0.0071	0.5420	0.0147	0.1012
High10	30	1.2914	1.3976	0.1532	-0.0549	-0.0551	1.6598	-0.0472	0.0815
High10	60	1.6169	1.8330	0.0086	-0.0495	-0.0485	2.3981	-0.0481	0.2903
Low20	30	0.9524	0.9361	0.1738	0.1458	0.1287	0.9859	0.1462	0.2583
Low20	60	0.9296	0.9203	0.1553	0.1457	0.1364	0.9848	0.1454	0.3685
High20	30	2.3486	2.7771	-0.6093	-0.0345	-0.0327	4.2958	-0.0872	0.2989
High20	60	2.3236	2.7563	-0.6592	-0.0341	-0.0338	4.2954	-0.0918	0.6122

Table 4: For each estimator, estimated relative root mean square errors based on 5,000 samples from population I(a), where the samples are contaminated with non-representative outliers.

Outliers	*n*	LS-1	LS-2	LS-3	MM-1	MM-2	L1-1	L1-2	CH
None	30	0.183	0.190	0.396	0.398	5.430	0.183	0.400	0.141
None	60	0.127	0.129	0.283	0.284	1.338	0.127	0.284	0.101
Low10	30	0.518	0.524	0.449	0.391	2.894	0.513	0.403	0.318
Low10	60	0.495	0.497	0.337	0.316	0.510	0.493	0.320	0.307
High10	30	0.824	1.201	1.014	0.307	2.881	1.504	0.588	0.191
High10	60	0.785	1.185	0.622	0.192	0.435	1.499	0.277	0.245
Low20	30	0.906	0.914	0.646	0.616	1.266	0.896	0.620	0.499
Low20	60	0.887	0.889	0.593	0.588	0.597	0.884	0.589	0.493
High20	30	1.087	1.719	1.488	0.276	1.126	0.940	1.687	0.289
High20	60	1.058	1.709	1.525	0.171	0.164	0.763	1.781	0.423

Table 5: For each estimator, estimated relative root mean square errors based on 5,000 samples from population II(a), where the samples are contaminated with non-representative outliers.

Outliers	*n*	LS-1	LS-2	LS-3	MM-1	MM-2	L1-1	L1-2	CH
None	30	0.102	0.101	0.286	0.312	0.270	0.102	0.291	0.210
None	60	0.068	0.068	0.233	0.242	0.233	0.068	0.235	0.145
Low10	30	0.446	0.449	0.235	0.228	0.233	0.436	0.222	0.175
Low10	60	0.433	0.434	0.189	0.196	0.197	0.428	0.194	0.119
High10	30	0.887	1.062	0.614	0.217	0.204	1.438	0.232	0.192
High10	60	0.869	1.051	0.398	0.153	0.147	1.434	0.162	0.207
Low20	30	0.800	0.805	0.460	0.461	0.451	0.784	0.458	0.154
Low20	60	0.786	0.789	0.463	0.466	0.457	0.779	0.464	0.132
High20	30	1.213	1.532	1.088	0.214	0.171	2.474	0.332	0.227
High20	60	1.200	1.524	1.148	0.186	0.131	2.473	0.330	0.355

Table 6: For each estimator, estimated relative root mean square errors based on 5,000 samples from population III(a), where the samples are contaminated with non-representative outliers.

Outliers	*n*	LS-1	LS-2	LS-3	MM-1	MM-2	L1-1	L1-2	CH
None	30	0.155	0.155	0.158	0.159	0.163	0.157	0.154	0.179
None	60	0.107	0.106	0.113	0.117	0.120	0.107	0.112	0.132
Low10	30	0.421	0.411	0.179	0.155	0.156	0.400	0.150	0.208
Low10	60	0.545	0.539	0.134	0.115	0.113	0.550	0.111	0.206
High10	30	1.347	1.427	0.592	0.159	0.158	1.664	0.158	0.200
High10	60	1.641	1.847	0.496	0.117	0.115	2.399	0.122	0.318
Low20	30	0.975	0.959	0.276	0.267	0.240	0.993	0.256	0.434
Low20	60	0.940	0.932	0.207	0.214	0.198	0.988	0.207	0.479
High20	30	2.385	2.799	1.198	0.174	0.160	4.297	0.225	0.360
High20	60	2.341	2.767	0.993	0.122	0.113	4.296	0.172	0.625

Table 7: For each estimator, estimated relative biases based on 5,000 samples from populations I(a) through I(e), II(a) through II(e), III(a), and IV(a).

Population	*n*	LS-1	LS-2	LS-3	MM-1	MM-2	L1-1	L1-2	CH
I(a): None	30	0.006	0.003	0.038	0.038	1.082	0.006	0.039	0.047
I(a): None	60	0.001	0.000	0.039	0.039	0.289	0.001	0.040	0.043
I(b): 5L	30	0.318	0.320	0.166	0.156	0.904	0.315	0.158	0.151
I(b): 5L	60	0.310	0.311	0.155	0.149	0.274	0.308	0.150	0.160
I(c): 5H	30	0.661	0.658	0.113	0.120	0.903	0.660	0.119	0.124
I(c): 5H	60	0.655	0.654	0.101	0.111	0.246	0.654	0.111	0.138
I(d): 10L	30	0.625	0.631	0.341	0.325	0.835	0.617	0.326	0.258
I(d): 10L	60	0.613	0.616	0.315	0.307	0.366	0.609	0.308	0.273
I(e): 10H	30	1.302	1.298	0.230	0.265	0.810	1.298	0.260	0.246
I(e): 10H	60	1.291	1.289	0.212	0.251	0.312	1.289	0.247	0.259
II(a): None	30	0.002	0.002	-0.192	-0.214	-0.189	0.001	-0.195	-0.024
II(a): None	60	0.001	0.001	-0.168	-0.174	-0.171	0.000	-0.169	-0.002
II(b): 5L	30	0.192	0.193	-0.040	-0.063	-0.056	0.187	-0.049	-0.018
II(b): 5L	60	0.188	0.188	-0.014	-0.020	-0.027	0.186	-0.017	0.012
II(c): 5H	30	0.418	0.417	-0.079	-0.091	-0.082	0.415	-0.079	0.031
II(c): 5H	60	0.415	0.415	-0.054	-0.050	-0.053	0.413	-0.049	0.071
II(d): 10L	30	0.383	0.386	0.124	0.100	0.102	0.374	0.111	-0.007
II(d): 10L	60	0.375	0.377	0.151	0.144	0.138	0.372	0.146	0.027
II(e): 10H	30	0.828	0.827	0.047	0.061	0.058	0.822	0.066	0.085
II(e): 10H	60	0.822	0.821	0.076	0.103	0.097	0.919	0.100	0.143
III(a): None	30	-0.001	-0.001	-0.058	-0.064	-0.067	0.001	-0.060	-0.022
III(a): None	60	-0.000	-0.001	-0.057	-0.064	-0.066	0.000	-0.060	-0.027
IV(a): 13Out	10	0.009	0.000	-0.035	0.006	-0.011	-0.021	-0.003	0.152
IV(a): 13Out	20	-0.017	-0.021	-0.059	0.005	-0.038	-0.034	-0.007	0.154
IV(a): 13Out	40	-0.031	-0.032	-0.072	0.004	-0.052	-0.038	-0.009	0.155

Table 8: For each estimator, estimated relative root mean square errors based on 5,000 samples from populations I(a) through I(e), II(a) through II(e), III(a), and IV(a).

Population	*n*	LS-1	LS-2	LS-3	MM-1	MM-2	L1-1	L1-2	CH
I(a): None	30	0.183	0.190	0.396	0.398	5.430	0.183	0.400	0.141
I(a): None	60	0.127	0.129	0.283	0.284	1.338	0.127	0.284	0.101
I(b): 5L	30	0.455	0.464	0.437	0.409	4.816	0.450	0.405	0.271
I(b): 5L	60	0.375	0.378	0.298	0.276	0.869	0.373	0.278	0.223
I(c): 5H	30	0.842	0.841	0.578	0.398	5.020	0.840	0.401	0.257
I(c): 5H	60	0.741	0.740	0.386	0.262	0.903	0.741	0.271	0.201
I(d): 10L	30	0.747	0.762	0.540	0.506	4.380	0.735	0.494	0.396
I(d): 10L	60	0.677	0.683	0.409	0.388	0.671	0.672	0.390	0.331
I(e): 10H	30	1.472	1.469	0.704	0.467	4.740	1.471	0.466	0.436
I(e): 10H	60	1.374	1.373	0.473	0.348	0.684	1.375	0.354	0.321
II(a): None	30	0.102	0.101	0.286	0.312	0.270	0.102	0.291	0.210
II(a): None	60	0.068	0.068	0.233	0.242	0.233	0.068	0.235	0.145
II(b): 5L	30	0.274	0.277	0.261	0.274	0.259	0.268	0.259	0.206
II(b): 5L	60	0.228	0.229	0.180	0.189	0.192	0.226	0.184	0.141
II(c): 5H	30	0.524	0.523	0.292	0.258	0.241	0.521	0.243	0.217
II(c): 5H	60	0.467	0.467	0.193	0.175	0.181	0.466	0.171	0.164
II(d): 10L	30	0.462	0.468	0.330	0.330	0.328	0.449	0.323	0.195
II(d): 10L	60	0.416	0.418	0.260	0.263	0.262	0.410	0.261	0.133
II(e): 10H	30	0.932	0.931	0.349	0.301	0.289	0.926	0.290	0.225
II(e): 10H	60	0.872	0.872	0.244	0.230	0.229	0.870	0.226	0.202
III(a): None	30	0.155	0.155	0.158	0.159	0.163	0.157	0.154	0.179
III(a): None	60	0.107	0.106	0.113	0.117	0.120	0.107	0.112	0.132
IV(a): 13Out	10	0.272	0.280	0.285	0.292	0.262	0.307	0.280	0.261
IV(a): 13Out	20	0.205	0.210	0.214	0.201	0.195	0.231	0.197	0.210
IV(a): 13Out	40	0.145	0.148	0.155	0.127	0.139	0.163	0.129	0.177