ELEC 321: Assignment #1.2

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Problem 1

1.
(a) $\frac{1}{57}$
(b) $\frac{8}{95}$
(c) $\frac{24}{95}$

2. $\mathrm{P}(A_i) = \dfrac{\binom{6}{i}\binom{14}{3-i}}{\binom{20}{i}}$, $i = 0, 1, 2, 3.$

3. 0.901

Problem 2

1. Follows from $\mathrm{P}(A) = \mathrm{P}(A \cap B) + \mathrm{P}(A^{\mathrm{c}} \cap B)$.
2. Follows from the result in part 1.

Problem 3

1. 0.98
2. 0.26
3. 0.848

Problem 4

1. $\binom{n}{j} p^j (1-p)^{n-j}$, $j = 0, 1, \ldots, n$.
2. $\dfrac{\binom{n-j+1}{j}}{\binom{n}{j}}$.
3. $\displaystyle\sum_{j \le \frac{n+1}{2}} \binom{n-j+1}{j} p^j (1-p)^{n-j}$.

Problem 5

(a)
(i) $\mathrm{P}(D|I_{10}) = 0.9672$.
(ii) $\mathrm{P}(D|I_{10}) = 0.0362$.
(iii) $\mathrm{P}(D|I_{10}) = 0.9673$.
(b) First part: follows from the theorem of total probability.
Second part: for $\alpha = 0.5$, $J(\alpha) \approx 0.005854$.
(c) $\alpha_{\text{opt}} = 0.59$; $J(\alpha_{\text{opt}}) = 0.0056$.

Problem 6

(a) mean = 50.5; variance = 833.25.
(b) $F_X(x) = 1 - (1 - \frac{x}{100})^5$; $f_X(x) = (1 - \frac{x-1}{100})^5 - (1 - \frac{x}{100})^5$.
From simulations: $E(X) = 17.17$; $\mathrm{Var}(X) = 198.36$.

Problem 7

$p \ge \frac{1}{3}$.

Problem 8

(a) If X denotes the cell index in which the ball lands, then $P(X = i) = \binom{4}{i} \left(\frac{1}{2}\right)^4$, $i = 0, 1, \ldots, 4$.

(c) $P(X = i) = \binom{99}{i} \left(\frac{1}{2}\right)^{99}$, $i = 0, 1, \ldots, 99$.