Key to Assignment 2.1

October 18, 2017

Problem 1:

(a) With $q = 0.95(1-p)^m + 0.1(1-(1-p)^m)$:
$\mathbb{E}(X_m) = 10(m+1-mq)$
$\mathrm{SD}(X_m) = 10\sqrt{(m+1)^2(1-q)+q-(m+1-mq)^2}$.

(b) $\mathbb{E}(T_j) = k\mathbb{E}(X_m)$ and $\mathrm{Var}(T_j) = \sqrt{k} \times \mathrm{SD}(X_m)$

(c) The one with the smallest value of $E(T_j)$.

Problem 2:

(a) 200

(b) 0.63

(c) 0.54

Problem 3:

(a) Argue that the two events are equivalent.

(b) 0.19, 0.01, 0.0028, 2.29×10^{-5}

(c) mean ≈ 179.9583, standard deviation ≈ 42.2175

Problem 4:

(a) Involving injuries: mean = 7, variance = 7. Not involving injuries: mean = 35, variance = 35.

(b) 0.2883

(c) 0.1535

(d) 16

Problem 5:

(A) $\hat{\mu} = 11.96$ and $\hat{\sigma}^2 = 0.21^2$

(B.1) –

(B.2) $\hat{\alpha} = 3243.6$ and $\hat{\lambda} = 271.2$

(B.3) –

Problem 6:

(a) –

(b) range: $[0, \infty)$

$$F_Y(y) = \begin{cases} e^{-0.5\lambda}\left(e^{-\lambda\sqrt{y}} + e^{\lambda\sqrt{y}}\right) & \text{if } 0 \le y \le \frac{1}{4} \\ 1 - e^{-\lambda(0.5+\sqrt{y})} & \text{if } y \ge \frac{1}{4} \end{cases}$$

Evaluate $F_Y(1/4)$. If $F_Y(1/4)$ is larger than $\frac{1}{2}$ then solve $e^{-0.5\lambda}\left(e^{-\lambda\sqrt{y}} + e^{\lambda\sqrt{y}}\right) = \frac{1}{2}$. If $F_Y(1/4)$ is smaller than $\frac{1}{2}$ then solve $1 - e^{-\lambda(0.5+\sqrt{y})} = \frac{1}{2}$. For the mean and standard deviation use simulation.

Problem 7:

(a) The system first gets stronger with time, up to age 5 and then becomes weaker with time after age 5.

(b) $F(y) = 1 - e^{-[(y-5)^3/3+125/3]}$

(c) $m = 0.0279$

Problem 8:

(a) 0.15866

(b) 0.8413447

(c) 0.15866

Problem 9:

$\mu = 101.54$ and $\sigma = 6.6037$

Problem 10:

(a) 0.7745375

(b) 0.8219608

Problem 11:
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Problem 12:
Refer to tutorial notes.