NUMERICAL EXAMPLE

		X_2				
		1	2	3	4	Sum
	0	0.10	0.10	0.10	0.10	0.40
X_1	1	0.05	0.10	0.10	0.05	0.30
	2	0.00	0.00	0.10	0.20	0.30
	Sun	0.15	0.20	0.30	0.35	1.00

CONDITIONAL DISTRIBUTION [$X_2 \mid X_1 = x_1$]

x_2	$f(x_2\|0)$	$f(x_2\|1)$	$f(x_2\|2)$
1	1/4	1/6	0.0
2	1/4	2/6	0.0
3	1/4	2/6	1/3
4	1/4	1/6	2/3
Mean	2.50	2.50	3.667
Variance	1.25	0.9167	0.2197

$$E(X_2|0)=\frac{1+2+3+4}{4}=2.5$$

$$E(X_2|1)=\frac{1+4+6+4}{6}=2.5$$

$$E(X_2|2)=\frac{3+8}{3}=3.667$$

$\leftarrow$ Verify this.

TWO-STEP AVERAGE

x_1	$E(X_2 \vert x_1)$	$f(x_1)$
0	2.5	0.4
1	2.5	0.3
2	3.667	0.3

$$E\left(X_2\right) = E\left[E\left(X_2|X_1\right)\right] = 2.5 \times 0.4 + 2.5 \times 0.3 + 3.667 \times 0.3 = 2.85$$

DIRECT CALCULATION OF $E(X_2)$

		X_2				
		1	2	3	4	Sum
	0	0.10	0.10	0.10	0.10	0.40
X_1	1	0.05	0.10	0.10	0.05	0.30
	2	0.00	0.00	0.10	0.20	0.30
	Sun	0.15	0.20	0.30	0.35	1.00

$$E(X_2) \;=\; 0.15 + 2 \times 0.20 + 3 \times 0.30 + 4 \times 0.35 = 2.85$$

TOTAL VARIANCE PARTITION

$$Var\left(X_2\right) = E\left(Var\left(X_2|X_1\right)\right) + Var\left(E\left(X_2|X_1\right)\right)$$

$$= \text{Unexplained} \;\; + \;\; \text{Explained}$$

UNEXPLAINED VARIANCE: $E\left(Var\left(X_2|X_1\right)\right)$

x_1	$Var(X_2|x_1)$	$f(x_1)$	$Var(X_2|x_1)f(x_1)$
0	1.25	0.4	0.50000
1	0.9167	0.3	0.27501
2	0.2198	0.3	0.06594
		1.0	**0.84095** $\searrow$
			$E\left(Var\left(X_2|X_1\right)\right)$

$$E\left(Var\left(X_2|X_1\right)\right) = 0.84095$$

EXPLAINED VARIANCE: $Var\left(E\left(X_2|X_1\right)\right)$

$$Var\left(E\left(X_2|X_1\right)\right) = E\left(E\left(X_2|X_1\right)^2\right) - \left[E\left(E\left(X_2|X_1\right)\right)\right]^2$$

$$= 8.409067 - 2.85^2 = 0.286567$$

x_1	$E(X_2\|x_1)$	$Var(X_2\|x_1)$	$f(x_1)$	$E(X_2\|x_1)f(x_1)$	$E(X_2\|x_1)^2 f(x_1)$
0	2.5	1.25	0.4	1.00	2.500000
1	2.5	0.9167	0.3	0.75	1.875000
2	3.667	0.2198	0.3	1.10	4.034067
			1.0	**2.85** $\searrow$	**8.409067** $\searrow$
				$E\left[E\left(X_2\|X_1\right)\right]$	$E\left[E\left(X_2\|X_1\right)^2\right]$

$$\% \text{ of Explained Variance} = \frac{0.2867665}{1.1275} 100\% = 25.43\%$$