ELEC 321 - STAT 357

MIDTERM, FALL 2015

Student Name (please print)______________________________

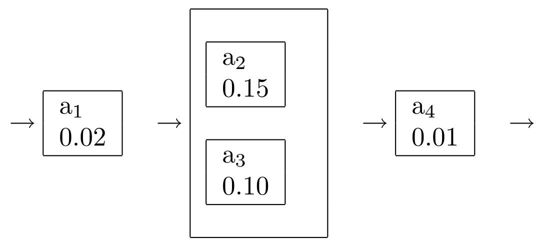
Student Number____________________________

Problem 1 [20]: Suppose that $P(A) = 0.20$, $P(B|A) = 0.50$ and $P(B|A^c) = 0.25$.

(a) [**10**] Calculate $P(A \cap B)$ and $P(A^c \cap B)$

(b) [**10**] Calculate $P(A|B)$.

Problem 2 [20]: Calculate the reliability of the following system of independent components. The probabilities of failure for each component are given in the diagram.



Problem 3 [20]: Suppose that a certain random event occurs with rate $\lambda = 12$ per day and that the assumptions for a Poisson process are satisfied.

(a) [**7**] Let W be the waiting time (in hours) until the occurrence of the next event. Calculate $P\,(W < 1/3)$.

(b) [**7**] Let Y be the number of occurrences of the random event in the next 6 hours. Calculate $P\,(Y \geq 2)$.

(c) [**6**] What is the expected number of occurrences in the next 2 days? What is the corresponding standard deviation?

Problem 4 [20]: Suppose U is a uniform random variable in the interval $(0, 1)$. Let

$$X = e^U$$

(a) [**3**] What is the range of X ?

(b) [**10**] Derive the distribution and density functions for X.

(c) [**7**] Calculate $E(X)$ and $Var(X)$.

Problem 5 [20]: The continuous random variable X represents the lifetime of a system in years. The failure rate function for the system is

$$\lambda(x) = ax, \quad \text{where } a > 0 \text{ is a given constant.}$$

(a) [10] Derive the distribution and density functions for X.

(b) [8] For which value of a is the median life of X equal to 15 [that is, $F(15) = 1/2$]?

(c) [8] Choose one: The system with lifetime X gets (i) Weaker ___ (ii) Stronger ___ (iii) Unchanged ___ with time. Please justify your answer.

Overflow

Overflow

Some Formulas

Poisson Random Variables:

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0, 1, 2, ...$$

$$E(X) = Var(X) = \lambda$$

Exponential Random Variables:

$$f(x) = \lambda e^{-\lambda x}, \quad x > 0$$
$$F(x) = 1 - e^{\lambda x}, \quad x > 0$$
$$E(X) = \frac{1}{\lambda}, \quad Var(X) = \frac{1}{\lambda^2}$$

Bayes' Formula

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B|A)\,P(A) + P(B|A^c)\,(1 - P(A))}$$