$\boxed{\text{Problem A.12}}$

majority decoding algorithms

Tx		Rx	
"0"		"1"	$P_1 = 1 - p$
		"0"	$P_0 = p$
"1"		"1"	$P_1 = p'$
		"0"	$P_0 = 1 - p'$

for simplicity we assume $p = p'$

Tx	Rx
"000"	$P(\text{0 received}) > p$

(a) $N = 2$, $2N + 1 = 5$, $N + 1 = 3$

To decode this bit correctly, at least 3 positions are received correcty on its own

$$P(X \geq 3) = \binom{5}{3}(0.8)^3(0.2)^2 + \binom{5}{4}(0.8)^4(0.2) + \binom{5}{5}(0.8)^5$$

$$\approx 0.942$$

(b) $2N+1 = 3, \quad N+1 = 2$

$$P(X \geq 2) = \binom{3}{2} p^2 (1-p) + \binom{3}{3} p^3 \geq 0.942$$

$$\Rightarrow 3p^2(1-p) + p^3 - 0.942 \geq 0$$

$$3p^2 - 3p^3 + 3p^3 - 0.942 \geq 0$$

$$-2p^3 + 3p^2 - 0.9408 \geq 0$$

$$\underline{p \geq 0.8536}$$

$\boxed{\text{Problem A. 13}}$

$\begin{cases} 13 \text{ ranks} \\ 4 \text{ types of cards} \end{cases}$

(a)

full house "AAABB"

$$P(\text{full house}) = \frac{\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}}{\binom{52}{5}} \approx 0.00144$$

(b) "AABBC"

$\nearrow$ pattern

$$P(\text{two pairs}) = \frac{\binom{13}{1}\binom{4}{2}\binom{12}{1}\binom{4}{2}\binom{11}{1}\binom{4}{1}}{\binom{52}{5}} \approx 0.047539$$

$\boxed{\text{Problem A.14}}$

$$P(X = \cancel{n}\, k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\frac{d}{dp} P(X=k) = k\binom{n}{k} p^{k-1}(1-p)^{n-k}$$
$$- (n-k)\binom{n}{k} p^k (1-p)^{n-k-1}$$

$$\frac{d}{dp} P(X=k) = 0$$

$$p = \frac{k}{n}$$

$\boxed{\text{Problem A.15}}$

$$P(X=k) = \frac{\lambda^k}{k!} e^{-\lambda}$$

$$\frac{d}{d\lambda} P(X=k) = k\frac{\lambda^{k-1}}{k!} e^{-\lambda} - \frac{\lambda^k}{k!} e^{-\lambda} = 0$$

$$\lambda = k$$

$\boxed{\text{Problem A.16}}$

The Poisson distribution

$$P(X=k) = \frac{\lambda^k e^{-\lambda}}{k!}, \quad k=0,1,2,3\ldots$$

mean : $\cancel{x}$ λ SD : $\sigma = \sqrt{\lambda}$

(a) $\lambda = 3 \times 2 = 6$

$$P(x=5) = \frac{6^5 e^{-5}}{5!} \approx 0.161$$

(b) $\lambda = \frac{3}{2} = 1.5$

$$P(X=k) = \frac{1.5^k e^{-1.5}}{k!}$$

$$P(x=0) = \frac{1.5^0 e^{-1.5}}{0!} \approx 0.223$$

(c) Distribution of time $\dfrac{e^{-\lambda t}\lambda^k t^{k-1}}{(k-1)!}$ for $t>0$

Let $k=1$, $e^{-\lambda t}\lambda$

$\lambda = 3$, $3e^{-3t}$ $E[x] = \frac{1}{\lambda} = \frac{1}{3}$ ~~hrs~~ hours

$Var[x] = \frac{1}{\lambda^2}$

(d) $(1-r_0)^3 + 3(1-r_0)^2 r_0 > P(\text{corret} \mid 0)$

$\uparrow$ majority decoding $\qquad \uparrow$ Sending one bit only

$r_0 < \frac{1}{2}$

(e) $\boxed{\text{Bayes's Formula}}$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)}$$

$P(0\text{sent} \mid 101 \text{ received})$

$$= \frac{P(0\text{ sent})P(101 \text{ received} \mid 0\text{sent})}{P(0\text{ sent})P(101 \text{ received} \mid 0\text{ sent}) + P(1\text{sent})P(101 \text{ received} \mid 1\text{ sent})}$$

$$= \frac{p * [1-(1-r_0)] * (1-r_0) * [1-(1-r_0)]}{p r_0^2 (1-r_0) + (1-p)(1-r_1)^2 r_1}$$

$$= \frac{p r_0^2 (1-r_0)}{p r_0^2 (1-r_0) + (1-p)(1-r_1)^2 r_1}$$

$\boxed{\text{Problem A.17}}$

(a) $P(\text{correct} \mid 0) = 1 - r_0$.

$P(\text{correct} \mid 1) = 1 - r_1$.

$P(\text{corret}) = P(\text{correct} \mid 0) * P(0) + P(\text{corret} \mid 1) * P(1)$

$= (1 - r_0)p + (1 - r_1) * (1 - p)$

(b) Since all the bits are independent

$P(1011 \text{ correct}) = P(1 \text{ corret})^3 P(0 \text{ correct})$

$= (1 - r_1)^3 (1 - r_0)$

(c) $P(0 \text{ decoded correctly}) = P(3 \text{ "0"s decoded}) + P(2 \text{ "0"s decoded})$

$= \binom{3}{3} [P(\text{correct} \mid 0)]^3 + C_3^2 [P(\text{correct} \mid 0)]^2 [1 - P(\text{correct})]$

$= (1 - r_0)^3 + 3(1 - r_0)^2 r_0$

Problem 7

Use Binomial Distribution

$$P(X=4) = \frac{5000!}{4!\,4996!}(0.001)^4(0.999)^{4996}$$

$$\begin{cases} n \text{ large} \\ \lambda = np \text{ small } (p<7) \end{cases}$$

~~P(2~~ $\lambda = np = (5000)(0.001) = 5$

$$P(4) \approx \frac{\lambda^4 e^{-\lambda}}{4!} = \frac{5^4 e^{-5}}{4!} \approx 0.175$$

Problem 8

Let x be the number of defectives in the sample.

$$P(\text{accept the lot}) = P(x \le 1) = P(0) + P(1)$$

$$= \frac{C_0^4 C_5^{16}}{C_5^{20}} + \frac{C_1^4 C_4^{16}}{C_5^{20}}$$

$$= \frac{\binom{0}{4}\binom{5}{16}}{\binom{5}{20}} + \frac{\binom{1}{4}\binom{4}{16}}{\binom{5}{20}}$$

$$\approx 0.7513$$