Stat 547 C
Some hints for Problem 4, Assignment 4

1) Argue that you can assume without loss of generality that the means are zero and the variances are 1.

Then w.l.g. assume that

$$\begin{pmatrix} X \\ Y \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}\right)$$

2) Notice that if

$$P(X>0, Y>0) = a$$

then

$$P(X>0, Y<0) = \frac{1-2a}{2} \quad \text{(why?)}$$

and so

$$E\{sign(X)\, sign(Y)\} = 4a - 1 = 4P(X>0, Y>0) - 1 \quad \text{(why?)}$$

Therefore, we just need to compute $P(X>0, Y>0)$.

3) Argue that we can assume w.l.g that $Y = \rho X - \sqrt{1-\rho^2}Z$, where $Z \sim N(0,1)$, independent from X.

4) Argue that

$$\begin{aligned} P(X>0, Y>0) &= P\left(X>0\ ,\ Z < \frac{\rho}{\sqrt{1-\rho^2}}X\right) \\ &= \frac{1}{2\pi} \int\int_{x>0, z<\left(\rho/\sqrt{1-\rho^2}\right)x} e^{-\frac{1}{2}\left(x^2+z^2\right)} dxdz \end{aligned}$$

5) Use polar coordinate to show that

$$P(X>0, Y>0) = \frac{1}{4} + \frac{1}{2\pi}\arcsin(\rho)$$

and so

$$E\{sign(X)\, sign(Y)\} = \frac{2}{\pi}\arcsin(\rho)$$