ASSIGNMENT 3

There are two parts to this assignment. The first part is on WeBWorK — the link is available on the course webpage. The second part consists of the questions on this page. You are expected to provide full solutions with complete justifications. You will be graded on the mathematical, logical and grammatical coherence and elegance of your solutions. Your solutions must be typed, with your name and student number at the top of the first page. If your solutions are on multiple pages, the pages must be stapled together.

Your written assignment must be handed in ***before your recitation on Friday, October 2****. The online assignment will close at* ***9:00 a.m. on Friday, October 2****.*

1. Find the values of a for which $\sum_{n\geq 1} \frac{a^{n-1}}{2^n}$ converges. For those values, what does the series converge to?

2. When applying the Comparison and Limit Comparison Tests, it is often useful to compare series to *p-series* of the form $\sum_{n\geq 1} \frac{1}{n^p}$ where $p > 0$. However, none of our convergence tests so far allow us to determine the convergence of the p-series themselves.

   In this question, we determine the convergence of p-series for $p \geq 2$ by proving a limited version of a result known as *Raabe's Test*. (It remains to determine the convergence for $1 < p < 2$, which we will do after we define the integral next term.)

   Let $\sum_{n\geq 1} a_n$ be a series with all positive terms, and $\lim_{n\to\infty} n\left(1 - \frac{a_{n+1}}{a_n}\right) = L > 1$.

   (a) Let r be such that $1 < r < L$. Prove that there exists some positive integer k such that
   $$(r-1)a_n < (n-1)a_n - na_{n+1} \qquad \text{for } n \geq k.$$
   (b) Let $l > k$ be an integer. Using part (a), prove that
   $$(r-1)\left(a_k + a_{k+1} + \cdots + a_l\right) < (k-1)a_k.$$
   (c) Using part (b) and the Monotone Convergence Theorem, prove that $\sum_{n\geq 1} a_n$ converges.

   (d) Using the result proven in the previous three parts, prove that $\sum_{n\geq 1} \frac{1}{n^p}$ converges for $p \geq 2$.

3. On your UBC Blog, give one real-life example for each of the following: a function, a sequence and a series. Your examples should be described in plain English. Then answer the following questions.
   (a) Does the graph of your function have a horizontal asymptote?
   (b) Does your sequence converge?
   (c) Does your series converge?

   Your post should be at least 150 words in length. It will be graded on clarity, originality and depth of understanding.

   On your assignment submission, please include the URL of your blog.