1. Find the equation of the line tangent to $y = (1 - 2x)^{10}$ at (0, 1).

We have $y' = -20(1-2x)^9$, which at x = 0 is equal to -20. Thus the equation of the line tangent to $y = (1-2x)^{10}$ at (0,1) is

$$y - 1 = -20x.$$

2. The curve described by the equation

$$(x^2 + y^2)^2 = x^2 - y^2 \tag{1}$$

is an example of a *lemniscate*.

- (a) Determine the points with nonzero x-values at which the lemniscate has horizontal tangent lines.
- (b) Can you conclude that the lemniscate has horizontal tangent lines at the origin? Can you conclude that it does *not* have horizontal tangent lines? Justify your answers in one or two paragraphs.

We differentiate (1) implicitly:

$$2(x^{2} + y^{2})(2x + 2yy') = 2x - 2yy';$$

and solving,

$$y' = \frac{x\left(1 - 2x^2 - 2y^2\right)}{y\left(1 + 2x^2 + 2y^2\right)}.$$
(2)

This vanishes when $1 - 2x^2 - 2y^2 = 0$ or x = 0.

When $1 - 2x^2 - 2y^2 = 0$, we have

$$x^{2} = \frac{1}{2} \left(1 - 2y^{2} \right). \tag{3}$$

Substituting this into (1) yields $y = \pm \frac{1}{2\sqrt{2}}$. Substituting these y-values into (3) yields $x = \pm \frac{\sqrt{3}}{2\sqrt{2}}$. Thus the lemniscate has horizontal tangent lines at the four points $\left(\pm \frac{\sqrt{3}}{2\sqrt{2}}, \pm \frac{1}{2\sqrt{2}}\right)$.

It remains to determine whether the lemniscate has horizontal tangent lines at the origin. It does not: for if y = 0, y' as described in (2) is undefined.

3. A small coffee cup from Great Dane Coffee, a campus coffee shop, is in the shape of a truncated right circular cone with large diameter 7.5 cm, small diameter 5.5 cm, and side length 9 cm, as pictured below.



Suppose coffee is poured in at a rate of $15 \text{ cm}^3/\text{sec.}$ What is the rate at which the depth of coffee is increasing when the cup is half full?

On your UBC Blog, post a related rates question, of similar or lower difficulty, which involves something that may be found on the UBC campus. Then post a solution to your question.

You will be graded on the appropriateness of your question and the correctness of your solution. Bonus marks will be given for particularly creative questions.

On your assignment submission, please include the URL of your blog.