ASSIGNMENT 8

There are two parts to this assignment. The first part is on WeBWorK — the link is available on the course webpage. The second part consists of the questions on this page. You are expected to provide full solutions with complete justifications. You will be graded on the mathematical, logical and grammatical coherence and elegance of your solutions. Your solutions must be typed, with your name and student number at the top of the first page. If your solutions are on multiple pages, the pages must be stapled together.

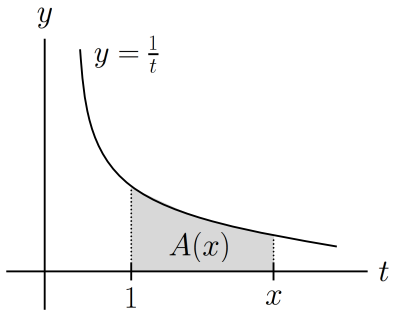
Your written assignment must be handed in ***before your recitation on Friday, November 13****. The online assignment will close at* ***9:00 a.m. on Friday, November 13****.*

1. Find a polynomial of degree 5 agreeing with $f(x) = \sin(x)$ and its first 5 derivatives at $x = 0$.

2. Note that $\left(\frac{1}{2}\right)^{1/2} = \left(\frac{1}{4}\right)^{1/4}$. Prove that there are infinitely many pairs of numbers $a < b$ such that $a^a = b^b$.

3. Recall that, in lectures, we introduced the functions e^x and $\log(x)$ in an intuitive but non-rigorous way. In this question, the function $\log(x)$ is defined in a rigorous way. A third, equally rigorous approach using power series will be considered next term.

   For positive x, let $A(x)$ be the area under the curve $y = \frac{1}{t}$ between $t = 1$ and $t = x$, as pictured below.



   Let $\log(x) = \begin{cases} -A(x) & \text{if } 0 < x \leq 1 \\ A(x) & \text{if } x > 1. \end{cases}$. Prove that $\frac{d}{dx}\log(x) = \frac{1}{x}$.

   (Hint: use the definition of derivative, and find lower and upper bounds for $A(x)$.)