ASSIGNMENT 9

Solutions

1. Find all the local extrema of $f(x) = 2x + \frac{2}{x}$.

   We have $f'(x) = 2 - \frac{2}{x^2}$, which vanishes when $x = \pm 1$. We determine the sign of $f'(x)$ between its critical points.

x	$(-\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
$f'(x)$	$+$	$-$	$-$	$+$
$f(x)$	increasing	decreasing	decreasing	increasing

   Thus $f(x)$ has a local maximum at $(-1, -4)$ and a local minimum at $(1, 4)$.

2. This question gives an example of a function illustrating the fact that the sign of a derivative *on an interval*, not a point, indicates whether the function is increasing or decreasing.

   Let $f(x) = \begin{cases} x + 2x^2 \cos\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$.

   (a) Prove that $f'(0) = 1$.

   (b) Prove that $f(x)$ is not increasing on any interval $(-\delta, \delta)$ where $\delta > 0$.

   We use the definition of derivative to find $f'(0)$:

$$f'(0) = \lim_{h \to 0} \frac{f(h) - f(0)}{h} = \lim_{h \to 0} \left(1 + 2h \cos\left(\frac{1}{h}\right)\right) = 1.$$

   To show that $f(x)$ is not increasing on any interval $(-\delta, \delta)$, we observe that, when $x \neq 0$,

$$f'(x) = 1 + 4x \cos\left(\frac{1}{x}\right) + 2 \sin\left(\frac{1}{x}\right).$$

   In particular, $f'\left(\frac{2}{(3+4n)\pi}\right) = -1$ for all integers n, which guarantees that there exists a value a in any interval $(0, \delta)$ for which $f'(a) = -1$. (There exist such values in $(-\delta, 0)$ as well.) Since the function $f'(x)$ is continuous on $(0, \delta)$, this means, by the Intermediate Value Theorem, that there exists an *interval* in $(0, \delta)$, centred on a, where $f'(x) < 0$ — and therefore $f(x)$ is decreasing — on that interval.

3. On your UBC Blog, post a question, on any topic covered in this course, which is suitable for the final exam. Then post a solution to your question.

   You will be graded on the appropriateness of your question and the correctness of your solution. You are encouraged to share your question on Piazza as a study resource for your classmates. Particularly good questions may be used on the actual exam.

   On your assignment submission, please include the URL of your blog.