ASSIGNMENT 1

There are two parts to this assignment. The first part is on WeBWorK — the link is available on the course webpage. The second part consists of the questions on this page. You are expected to provide full solutions with complete justifications. You will be graded on the mathematical, logical and grammatical coherence and elegance of your solutions. Your solutions must be typed, with your name and student number at the top of the first page. If your solutions are on multiple pages, the pages must be stapled together.

Your written assignment must be handed in before your recitation on Friday, January 15. The online assignment will close at 9:00 a.m. on Friday, January 15.

1. A function f(t) is odd if f(t) = -f(-t) for all t in the domain. For example, t^3 is an odd function. Let f(t) be an odd function that is defined everywhere. Prove using the definition of integral that, if f(t) is integrable on all intervals, then

$$\int_{-a}^{a} f(t) \, dt = 0$$

for all a.

2. Let

$$f(t) = \begin{cases} \frac{1}{q} & \text{if } t \text{ is a rational number } \frac{p}{q}, \text{ where } q > 0 \text{ and the number is written in lowest terms} \\ 0 & \text{if } t \text{ is an irrational number} \end{cases}$$

This function is differentiable nowhere, and continuous at exactly all irrational numbers. However, it turns out that it is integrable.

Prove that f(t) is integrable on [0, 1]. (Hints: use the definition of limit; and, given a positive integer N, why are there only finitely many x-values in [0, 1] such that $f(t) > \frac{1}{N}$?)

3. A function f(t) has a *removable discontinuity* at a if $\lim_{t \to a}$ and f(a) are defined but not equal. For example, the function

$$f(t) = \begin{cases} 0 & \text{if } t = 1, 2, 3\\ t^2 & \text{otherwise} \end{cases}$$

has removable discontinuities at 1, 2 and 3.

On your UBC Blog, give an argument in plain English why functions with finitely many removable discontinuities are integrable. You may assume without proof that continuous functions are integrable.

Your post should be at least 150 words in length. It should be as non-technical as possible. It will be graded on clarity and depth of understanding.

On your assignment submission, please include the URL of your blog.