## **ASSIGNMENT 4**

There are two parts to this assignment. The first part is on WeBWorK — the link is available on the course webpage. The second part consists of the questions on this page. You are expected to provide full solutions with complete justifications. You will be graded on the mathematical, logical and grammatical coherence and elegance of your solutions. Your solutions must be typed, with your name and student number at the top of the first page. If your solutions are on multiple pages, the pages must be stapled together.

Your written assignment must be handed in before your recitation on Friday, February 5. The online assignment will close at 9:00 a.m. on Friday, February 5.

- 1. Evaluate  $\int_{9}^{10} \frac{\sqrt{t}}{t^2 t} dt$ .
- 2. Let  $f_a(t) = (\log(a))^{\log(a)} t \sin(at)$  be defined for all a > 1. The graph of this function is pictured below.



Find a such that the shaded area under the first part of the curve is the smallest possible.

3. Let f(t) be a function such that  $f(t), f'(t), f''(t), \ldots, f^{(n+1)}(t)$  exist, and are continuous, on an interval containing a. Then for any x in that interval, Taylor's Theorem (with integral remainder) states that

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots + \frac{f^n(a)}{n!}(x-a)^n + \frac{1}{n!}\int_a^x f^{(n+1)}(t)(x-t)^n dt.$$
 (1)

In this question, you will prove and apply this theorem.

(a) Explain why equation (1) is true in the case n = 0; that is, explain why

$$f(x) = f(a) + \frac{1}{0!} \int_{a}^{x} f^{(0+1)}(t)(x-t)^{0} dt.$$

(Recall that we define 0! = 1.)

- (b) Suppose equation (1) is true in the case n = k. Prove that it is true in the case n = k + 1. (Hint: use integration by parts on the last term.)
- (c) Explain in one or two sentences why parts (a) and (b) imply that Taylor's Theorem is true.