

ASSIGNMENT 7

There are two parts to this assignment. The first part is on WeBWorK — the link is available on the course webpage. The second part consists of the questions on this page. You are expected to provide full solutions with complete justifications. You will be graded on the mathematical, logical and grammatical coherence and elegance of your solutions. Your solutions must be typed, with your name and student number at the top of the first page. If your solutions are on multiple pages, the pages must be stapled together.

Your written assignment must be handed in **before your recitation on Friday, March 18**. The online assignment will close at **9:00 a.m. on Friday, March 18**.

1. (a) Use a linear approximation to estimate $\log(1.01)$.
(b) Is your approximation $L(1.01)$ an underestimate, an overestimate, or equal to $\log(1.01)$? Justify your answer.
(c) Bound the size of the error $E(1.01) = \log(1.01) - L(1.01)$ using an appropriate theorem (and not using a calculator).

2. Let $f(x) = \begin{cases} e^{-1/x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$.

- (a) Prove that $f^{(n)}(0) = 0$ for all n .
(b) Explain why the Maclaurin series for $f(x)$ does not converge to $f(x)$ on any open interval.
3. In the previous question, you proved that the given function is not *analytic* at 0. On the other hand, the function e^x is analytic at 0 (and indeed everywhere). On your UBC Blog, explain in one or two paragraphs, with the use of examples, what it means for a function to be *analytic* at c , and why this is a useful definition. In particular, what distinguishes a function that is infinitely differentiable from one that is analytic?

Note that you will likely begin with a search on Wikipedia, Google, or in the index of a textbook, but that you must paraphrase what you find in a way that is comprehensible to someone with your background in calculus. You should avoid jargon and focus on the logical structure of what you write.

On your assignment submission, please include the URL of your blog.