## **ASSIGNMENT 9**

There are two parts to this assignment. The first part is on WeBWorK — the link is available on the course webpage. The second part consists of the questions on this page. You are expected to provide full solutions with complete justifications. You will be graded on the mathematical, logical and grammatical coherence and elegance of your solutions. Your solutions must be typed, with your name and student number at the top of the first page. If your solutions are on multiple pages, the pages must be stapled together.

Your written assignment must be handed in before the lecture on Monday, April 4. The online assignment will close at 9:00 a.m. on Friday, April 8.

1. The *Gompertz differential equation* is a version of the logistic differential equation also used to model population growth. It is

$$\frac{dP}{dt} = rP\log\left(\frac{K}{P}\right)$$

where r and K are constants. Solve the differential equation, assuming r = 0.05, K = 1000 and the initial condition P(0) = 500. Then sketch the graph of your solution. (You do not have to justify the shape of the graph.)

2. The *Bessel equation* is a differential equation which arises in problems of wave propagation. It is given by

$$x^{2}\frac{d^{2}y}{dx^{2}} + x\frac{dy}{dx} + (x^{2} - 1)y = 0.$$
 (1)

In this question, you will find one nontrivial solution to the Bessel equation using power series.

(a) Let  $y = \sum_{n \ge 0} a_n x^{\mu+n}$  be a solution to (1). Substitute this into the left-hand side of the Bessel equation,

and simplify into a single power series. (Your answer should use the summation notation " $\sum$ ", but you may find it helpful to write the first two terms separately.)

- (b) Let  $a_0 = 1$ . Find a pattern to describe  $a_1, a_2, a_3, a_4, \ldots$
- (c) Prove that the series y, with the coefficients as described in part (b), converges for all x.
- 3. In the first assignment of MATH 100, we stated that the exercise of posting mathematical reflections on your blog was to "train your ability to explain abstract mathematical ideas, a key skill in higher-level mathematics".

Pick any mathematical idea you learned about in MATH 100 or MATH 101, from limits to differential equations, or even a precalculus idea that was used. Then explain it on your blog in one to three paragraphs. Imagine you are introducing someone to the idea for the first time. Your explanation should be in plain English, it should avoid jargon and mathematical notation, and it must be different from any explanation that we gave to you in MATH 100 or MATH 101.

On your assignment submission, please include the URL of your blog.