

Case Study on Propagation of Variation: Aircraft Turbine

Jerome Sacks and William J. Welch

National Institute of Statistical Sciences and University of British Columbia

Adapted from materials prepared by Jerry Sacks and Will Welch for
various short courses

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Outline of Topics

You have fitted a Gaussian process (GP) model and have \hat{y} (i.e., $\hat{m}(\mathbf{x})$ from Module 3). **What's next?**

Use \hat{y} instead of $y(\mathbf{x})$ to **answer scientific and engineering questions.**

- 1 Science and Engineering Objectives
- 2 Application
- 3 GP Model Fitting
- 4 Visualization
- 5 Propagation of Variation



Some Science and Engineering Questions

- **Visualization**: What do the $y(\mathbf{x})$ input-output relationships look like?
- **Sensitivity analysis / screening**: What are the important variables?
- **Optimization**: What values of \mathbf{x} maximize/minimize y ? (Could have multiple output variables to optimize simultaneously.)
- **Propagation of variation**: If \mathbf{x} has a known distribution, what is the distribution of $y(\mathbf{x})$?
- ... other questions about $y(\mathbf{x})$

We are assuming $y(\mathbf{x})$ is too expensive to compute many times to answer such questions, so ...

- **Replace $y(\mathbf{x})$ with \hat{y} .**



Aircraft Turbine Application

Engineering objective: Design a component of a turbine blade assembly to meet fatigue time requirements.

Manufacturing variability at places on the assembly **propagates to variability in output** (stress at key points). Understanding this is essential to setting acceptable tolerances for the input variables.

How the GP models are used here: To estimate the distribution of each output variable by propagating a known \mathbf{x} distribution through \hat{y} .



Input Variables

Input variable	Description	Min (in)	Max (in)
RetArm	Retaining arm interference fit	-0.0055	0.0045
CPRabbit	Cooling-plate rabbit interference fit	-0.007	-0.003
CPDrop	Cooling-plate drop	-0.01	0.00



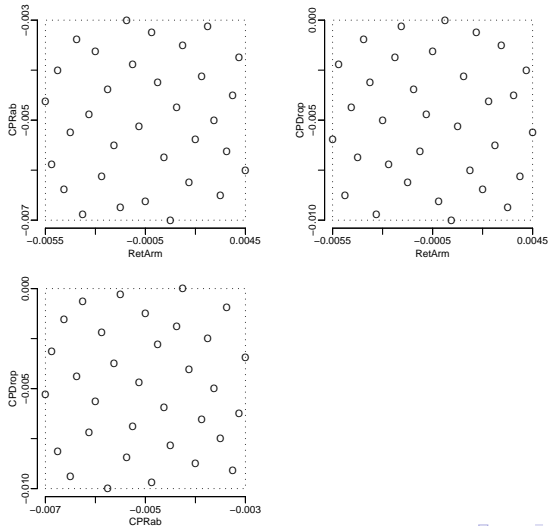
Experimental Design

- 33-run maximin Latin hypercube design
- It takes about 1 hour to run the computer code for one observation.



Experimental Design

33-Run Maximin Latin Hypercube Design



Output Variables

Output variable	Description
RabFilMean	Rabbit fillet mean stress
RabFilAlt	Rabbit fillet alternating stress
SWGFilMean	Seal wire groove mean stress
SWGFilAlt	Seal wire groove alternating stress



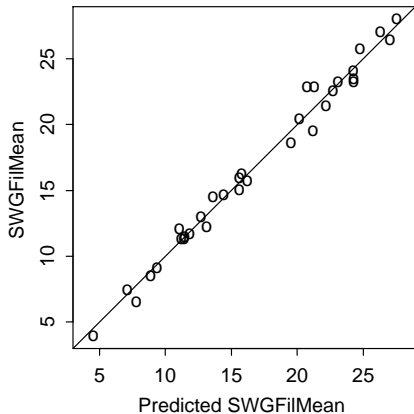
Fitting the GP Models: Summary

- Maximum likelihood estimation
- Cross-validation diagnostics look good
- Visualization shows some interesting nonlinear effects

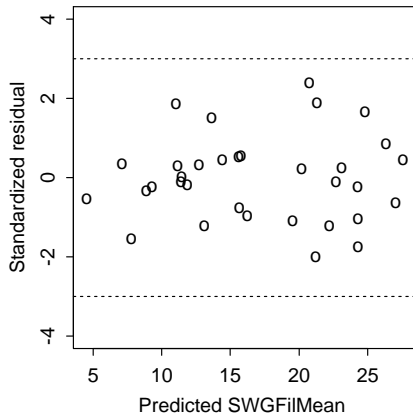


Diagnostics for SWGFilMean

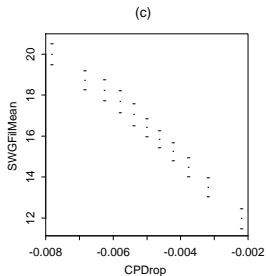
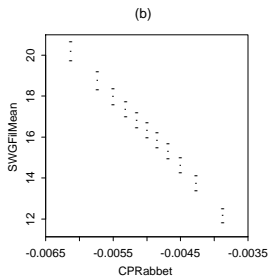
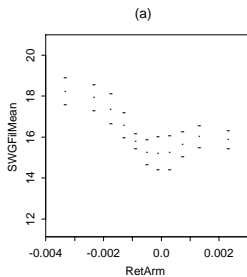
(a)



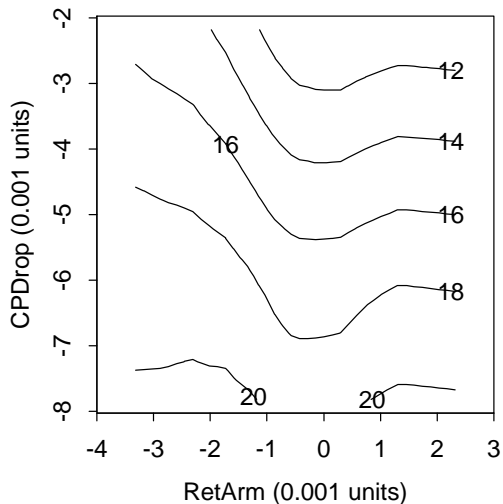
(b)



Estimated Main Effects for SWGFilMean



Estimated Joint Effect for SWGFilMean



Propagation of Variation

- **The manufacturing distributions** for x_1, \dots, x_3 (RetArm, CPRabbit, CPDrop) are
 - **independent**
 - **normal**
 - with **means** at the midpoints of the input ranges
 - and **standard deviations** 1/12 of the input ranges
 - e.g., x_1 (RetArm) is distributed $N(-0.0005, (0.01/12)^2)$.
- Take about **10,000 Monte Carlo** samples from the manufacturing distributions.
- Compute the corresponding \hat{y} values (cheap!)
- Plot the \hat{y} distributions to achieve the **engineering objective of estimating the propagated variation**
- e.g., output variable SWGFilAlt ...



Estimated Distribution of SWGFiAlt

